The authors prove the following unusual generalization of Kneser’s theorem on the cardinality of sumsets in commutative groups. Let $E \subset K$ be fields, $A, B$ finite dimensional subspaces of $K$ over $E$. Assume that every algebraic element in $K$ is separable over $E$. Then
\[ \dim AB \geq \dim A + \dim B - \dim H, \]
where $\dim$ means dimension over $E$ and $H = \{x \in K : xAB \subset AB\}$.

The connection with Kneser’s theorem is not obvious; the authors use a new (at least to the reviewer) method to embed a group into a field so that cardinalities of subsets are transformed into dimensions of subspaces.

It is not yet clear whether this approach leads to new advances in additive number theory, but it presents a refreshing new look at some classical methods and results.

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