Math 617 Potential Projects  
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These are projects. To my knowledge, a complete solution to these questions is not available though you will find progress on these problems and related problems in books and journals. A successful project report need not solve these problems. In fact, substantial progress is likely to be publishable. A good project report must identify the relevant issues and difficulties, and address them.

1. Let $L$ be a regular, self-adjoint Sturm-Liouville operator:

$$L[u] = -(p(x)u'(x))' + q(x)u(x),$$

on $0 \leq x \leq 1$ with boundary conditions

$$\alpha_1 u(0) + \alpha_2 u'(0) = 0$$

$$\beta_1 u(1) + \beta_2 u'(1) = 0.$$  

If $p$ and $q$ are $C^\infty$, then we can seek weak solutions to the inhomogeneous problem $Lu = f$. When can these restrictions on $p$ and $q$ be relaxed? Must further restrictions be placed on $f$?

2. Classify all solutions to the system

$$-\frac{1}{2} \vec{x} \cdot \nabla \omega + U \frac{\partial \omega}{\partial x} = \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

where $U$ is a real parameter, $\omega(x, y)$ $\psi(x, y)$ are smooth (perhaps analytic) functions and satisfy the constraint that $\nabla \omega$ and $\nabla \psi$ are parallel to one another, and

$$\lim_{|x| \to \infty} \omega = 0.$$  

Notice that one can solve the first and second PDE’s using standard methods. The hard part is satisfying the constraint and BC’s simultaneously.