Altea’s Passport™ System: Electrical and Thermal Model

Introduction:

The Passport System (Figure 1) allows transdermal drug delivery by burning small holes in the outer layer of the skin. This can be accomplished by supplying a voltage to the array of filaments (Figure 2) such that it achieves a desired temperature profile. Our goal is to design a voltage profile that will be delivered to the array of stainless steel filaments such that the filaments achieve an optimal temperature profile without exceeding 700 degrees Fahrenheit. This temperature was determined by the amount of energy required to vaporize the outer layer of skin without penetrating far enough to reach the nerves. The difficulty of this project lies in the complexity of the mathematical model. To obtain an accurate solution the model must account for three different modes of heat transfer, one of which necessitates a spatially variant model. Furthermore, several variables in the model that are often assumed to be constants vary greatly during the process because of the wide range of temperatures encountered by the filaments. Our ability to successfully account for each of the modes of heat transfer and to obtain reasonably accurate values for certain variables will ultimately determine the quality of our model.

Background:

![Figure 1: Passport System](image)

Altea Therapeutics is a private company based in Georgia that specializes in medical technology. It has designed the PassPort™ system, which is a noninvasive drug delivery system for the short or long term delivery of peptides, proteins, small-molecule drugs, genes and vaccines. Before this system, the delivery of medicine through the skin was limited to lipid-soluble drugs with relatively small molecular weights. This restriction was due to the properties of the stratum corneum, the outermost layer of the skin. Since the stratum corneum is about a 15µm waterproof layer of dead skin, it keeps out harmful bacteria as well as beneficial drugs. Hence, the PassPort™ system was designed to allow a greater variety of drugs to be delivered through the skin.
The PassPort™ system works for a wide range of drugs through poration of the stratum corneum. A hand-held reusable battery-driven activator heats an array of filaments, which in turn creates micropores of the skin by vaporizing microscopic amounts of dead cells from the stratum corneum. A patch containing medicine is then placed over the micropores, which are deep enough to allow the entry of the drugs into the skin, but shallow enough that the pain receptors in the dermis, the layer of skin below the stratum corneum, are unaffected. Thus, the whole method is painless.

Since the filaments are measured in micrometers, the production of the array must be precise. First, a copper rectangle is electroplated onto a stainless steel #304 rectangle of the same size. Then some of the copper is etched away, so a zigzagging “river” of stainless steel is left in the middle of the copper and steel-layered rectangle. The copper spokes that remain are called traces. Then a triple YAG laser is used to cut out some pieces of the stainless steel in the "river" so that only the stainless steel filaments remain.

The circuitry of the PassPort™ system involves 80 stainless steel filaments (Figure 2) in parallel, a voltage source with a resistance of about 4 ohms, and a 10 Farad capacitor. The copper traces act as the anode and cathode of this system and are connected by the filaments, which close the circuit.

More information can be found on Altea’s website at http://www.alteatherapeutics.com/main.htm

**Constraints and Measurements:**

Material: Filaments are made out of stainless steel and are held in place by copper traces
Geometry: There are 80 filaments, each with dimensions of 500µm by 50µm by 15µm.
Electrical Current: The maximum current is 1000 amps.
Temperature: The maximum allowable temperature is 700° Fahrenheit.
Time: A single temperature profile occurs in 2 milliseconds with 1 second cooling time.

Developing the Model:

We are currently developing a mathematical model that represents the electrical and thermal models for Altea’s Passport system. Since electrical conductivity, $\sigma(T)$, which depends on resistance, $R(T)$, is included in both the electrical and thermal models, we arrive at a coupled system.

In the following coupled system, the temperature with respect to time will be our focus. For initial analysis and simplicity, we will assume no spatial dependence.

\begin{equation}
V(t') = I(t')R(T)
\end{equation}

\begin{equation}
mc_p \frac{dT}{dt} = I(t')^2 R(T) - (q_{\text{conv}} + q_{\text{rad}} + q_{\text{cond}})
\end{equation}

Note that equation 1 is Ohm’s Law where $V(t')$ is voltage, $I(t')$ is current, and $R(T)$ is resistance, and equation 2 is based on energy conservation and energy transfer per unit time. Therefore the units for each term in equation 2 shall be energy per time or Joules/second.

Since $R(T) = \frac{L}{\sigma(T)A}$ where L is the length of the material and A is the cross-sectional area of the material, we can adjust equation 1 to the following:

\begin{equation}
V(t') = I(t')L \frac{A}{\sigma(T)A}
\end{equation}

Since equation 3 is easy to manipulate, we can solve for current and substitute it into equation 2. This results in the following mathematical model

\begin{equation}
mc_p \frac{dT}{dt} = \frac{V(t')^2 \sigma(T) A}{L} - (q_{\text{conv}} + q_{\text{rad}} + q_{\text{cond}})
\end{equation}

Going from left to right, this model explains that the energy storage with respect to time in the system is equal to the heating of the system due to the Joule heating phenomenon minus the energy losses of the system. These losses include energy losses due to natural convection to the surrounding air, radiation losses to other objects, and finally conductive losses to the copper anode and cathode. These losses will be discussed in more detail in following paragraphs.
The left hand side of equation 4

\[ mc_p \frac{dT}{dt} \]  

(5)

is the energy storage term in the system, where \( m \) is mass, \( c_p \) is specific heat, and \( \frac{dT}{dt} \) is the temperature differential with respect to time.

The first term on the right hand side of equation 4

\[ \frac{V(t')^2 \sigma(T)A}{L} \]  

(6)

is the only energy generation term present in our model. It explains the heat input per unit time as a function of time and temperature. This heat is generated by the phenomenon of Joule heating and is governed by Joule’s Law,

\[ Q = Pt = I^2 Rt = I^2 \frac{L}{\sigma A}t \]  

(7)

Joule’s Law describes the amount of heat, \( Q \), generated when a current, \( I \), flows through a conductor of length, \( L \), cross sectional area, \( A \), and conductivity, \( \sigma \), for a time, \( t \). The data provided by Altea indicates that resistance is an exponential growth function; however, for simplicity, we can approximate resistance, and therefore conductivity, to have a linear relationship with temperature. Our initial approximation for the resistance of stainless steel #304 is:

\[ R(T) = 5 \times 10^{-11} \frac{\Omega}{K} \cdot T \]  

(8)

The next three terms in equation 4 explain the various modes in which heat is dissipated from the filament. The first heat loss term governs the heat that is dissipated due to natural convection to the surrounding air. The equation is as follows:

\[ q_{\text{conv}} = hA_s (T_s - T_\infty) \]  

(9)

where \( h \) is the coefficient of convection, \( T_s \) is the temperature of the surface of the filament, \( A_s \) is the surface area, and \( T_\infty \) is the temperature of the ambient air.

The next heat dissipation term explicates the energy lost due to radiation to the surroundings and can be seen below.

\[ q_{\text{rad}} = \varepsilon \sigma_{SB} A_s \left( T_s^4 - T_{\text{air}}^4 \right) \]  

(10)
In this case $\varepsilon$ is the emissivity of the material, $A_s$ is the surface area, $\sigma_{SB}$ is Stefan-Boltzmann’s constant ($\sigma_{SB} = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$), $T_s$ is the temperature of the surface of the filament, and $T_{sur}$ is the temperature of the surrounding objects which can be approximated by the temperature of the ambient air. Note that the Stefan-Boltzmann’s constant is usually denoted by $\sigma$, but in order to avoid confusion with conductivity, $\sigma(T)$, we have adjusted the notation to $\sigma_{SB}$.

The final heat dissipation term takes into account the conduction of heat from the ends of the filaments to the copper electrodes. This term has not yet been fully explored and is expected to be the most difficult heat loss term to model mathematically because it requires spatial variance. In addition, the energy losses modeled by this term are expected to be significant because of the large temperature gradient between the copper electrodes and the steel filaments and because of the massive heat capacity of the copper electrodes relative to the filaments. This was confirmed visually in our experiment with metal blocks and nichrome wire. It was clear that the wire temperature close to the blocks was significantly lower than the middle of the wire.

To account for spatial variance, equation 4 can be improved generating the following partial differential equation

$$\rho c_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial x^2} + \left[ \nabla \psi \right]^2 \frac{1}{r(T)}$$

where $\rho$ is density, $\psi$ is electric potential and $r(T)$ is resistivity. Note that the heat losses are not including in the PDE because they will be incorporated into the boundary conditions. This equation is the basis for our future mathematical model and will be further developed in the weeks to come.

**Scaling the Model:**

The purpose of scaling a model is to allow comparison of a broad range of applications. The first step is to non-dimensionalize our variables in order to reduce parameter space.

$$t = \frac{t'}{t_p}$$

$$U = \frac{T - T_{x}}{T_{m} - T_{x}}$$

The scaling variable for time $t_p$ is the time elapsed during a voltage pulse. The scaling variables for temperature are $T_m$, the maximum temperature, and $T_{x}$, the temperature of
the ambient air. We can now rewrite the functions in our model using the non-dimensional variables.

\[
\sigma(T) = \sigma_o f \left( \frac{T - T_c}{T_m - T_c} \right)
\]

\[\sigma_o \] is the conductivity of the material at room temperature, so \( \sigma(T_c) = \sigma_o \) and \( V_m \) is the maximum voltage reached in the pulse.

\[
V(t')^2 = V_m^2 g \left( \frac{t'}{t_p} \right)
\]

Now that we have non-dimensionalized the parameters, we can plug them back into a simplified version of equation 4. In this simplification we will ignore cooling effects from conduction. The simplified scaled model is as follows:

\[
U_t = Pf(U)g(t) - BU - B_r \left[ U + \left( \frac{T_c}{T_m - T_c} \right) \right]^4 - \left( \frac{T_c}{T_m - T_c} \right)^4
\]

\[
P = \frac{t_p V_m^2}{mc_p (T_m - T_c) R_o}
\]

\[
B = \frac{hA_s t_p}{mc_p}
\]

\[
B_r = \frac{\varepsilon \sigma B TB}{mc_p}
\]

An initial condition is required to solve this equation, it is given below.

At time equals zero: \( T(0) = T_c \)
therefore \( U(0) = 0 \)

To show the relevance of radiation in comparison to convection, the radiation term is scaled so that it is in a non-dimensionalized form (equation 19). The emissivity of a body is a property that measures how efficiently it radiates its energy. This value can range between 0 and 1 and is strongly dependant on the surface material and finish. For most stainless steels in the appropriate temperature range, the values of emissivity range from about .17 to .28. See Introduction to Heat Transfer by Frank P. Incropera and David P. DeWitt for more specific values. When comparing this \( B_r \) to the \( B \) of convection, we see that their ratio shows us their relative significance. The ratio of \( B \) to \( B_r \) is as follows:
\[ h : \varepsilon \sigma_{SB} (T_m - T_\infty)^3 \]  \hspace{1cm} (22)

Since these two values are approximately of the same order, they are both equally important heat loss terms.

**Solving the Model:**

As a further simplification, we reduce equation 16 to ignore radiation, which allows us to arrive at an exact solution. The model is shown below.

\[ U_t = Pf(U)g(t) - BU \]  \hspace{1cm} (23)

We have assumed that conductivity varies linearly versus temperature, and therefore the function \( f(U) \) is linear and can be written as:

\[ f(U) = C_1 U + C_2 \]  \hspace{1cm} (24)

Note that \( f(0) = C_2 \) and since \( \sigma(T_\infty) = \sigma_0 f(0) = \sigma_0 \), \( C_2 = 1 \). We can rewrite equation 24 as

\[ f(U) = C_1 U + 1 \]  \hspace{1cm} (25)

The linearity of \( f(U) \) provides us with an exact solution to equation 23.

\[ U = \frac{P \int_0^t g(z) \cdot e^{\int BPCg(t)dt}dz}{e^{\int BPCg(t)dt}} \]  \hspace{1cm} (26)

The strength of this model is that it can provide us with an exact solution. Its weaknesses are that it does not take into account spatial variation, conduction, and radiation. When we include the conduction and radiation terms in our model, we will have to solve it numerically because of its non-linearity.

Using equation 26, our goal is to optimize the voltage function \( g(t) \). To begin we will choose an optimum temperature profile based on the information that Altea has provided. Possible choices for a temperature profile include a plateau at 700° F, a peak at 700° F, and a profile that minimizes rise time. Once we choose our optimal temperature profile \( T(t) \), we can compare it to the temperature profile \( U(t) \) given from equation 26 when different voltage pulses are used. Our goal is to minimize the difference between these two temperature profiles. We can quantify the difference with the following equation:
Solutions with Specific Voltage Profiles:

Now that we have a general solution to our simplified model, we can prescribe a specific voltage profile to get a specific solution. Then we can compare this theoretical solution with experimental data to test the validity of our model.

The simplest voltage profile is a constant voltage. Since the voltage is scaled, we get the Heaviside function below:

\[ g(t) = 1 \quad \text{for} \quad 0 \leq t \leq 1 \quad (28) \]

\[ g(t) = 0 \quad \text{for} \quad t > 1 \quad (29) \]

We substitute equation 28 into the general solution and get

\[ U(t) = \frac{P}{B - PC_1} \left( 1 - e^{-(B - PC_1)t} \right) \quad (30) \]

Note that this equation holds for \( 0 \leq t \leq 1 \).

Substituting equation 29 into the general solution gives us that

\[ U(t) = \left( \frac{U(1)}{e^{-Bt}} \right) e^{-Bt} \quad (31) \]

This equation holds for \( t > 1 \).

The graph of the solution depends on the relationship between \( B \) and \( PC_1 \). If \( B > PC_1 \), then the solution for \( 0 \leq t \leq 1 \) has an exponential decay term. If \( B < PC_1 \), then the solution for \( 0 \leq t \leq 1 \) had an exponential growth term. A general graph of the solution can be seen below for both cases.
We must also examine when \( B = PC_1 \). Again by examining equation 26, we can see that the solution is

\[
U(t) = Pt
\]  

(32)

**Experiments:**

**Experiment 1:**

The goal of the first experiment is to obtain an approximate value for the coefficient of convection that is used in equation 10. To validate the value of \( h \) that we obtain from the experiment we will compare it to a value of \( h \) calculated theoretically. To simplify our experiment we will disregard radiation and conduction along the wire. The joule heating term can also be eliminated from the equation because in the experiment we will not begin measuring the temperature change of the wire until after the voltage is stopped.

Using these assumptions, our model is simplified to the following:

\[
m_c \rho \frac{dT}{dt} = -hA_x(T - T_\infty)
\]  

(33)

\[
T(0) = T_{max}
\]  

(34)

Using the temperature scale from equation 13, and the time scale

\[
t = \frac{t'hA_x}{mc_\rho}
\]  

(35)

we get the following scaled solution:

\[
U = e^{-t}
\]  

(36)

Then we take the log of equation 36 to get
\[
\ln(U) = -t = -\frac{t' h A_s}{mc_p}
\]  

(37)

We use this equation with experimental data to find \( h \).

To setup the first experiment we first attach the ends of the wire to two aluminum blocks that serve as the anode and cathode of the electrical circuit. The wire is made of nichrome, is .042 meters in length, and has a radius of \( 4.56 \times 10^{-4} \) meters. Voltage is applied to the wire elevating its temperature to a set value. The voltage is then removed and the wire is allowed to cool. Measurements are taken of the temperature of the wire versus time after the voltage is removed. We scale the temperature measurements and take the log of them. Then we find the slope of the resulting line, so we can solve for \( h \). This experiment is performed twice with different values of the maximum temperature of the wire. When the maximum temperature reaches 307.5 K, the resulting \( h \) is 95.5 watts per meter squared Kelvin. When the maximum temperature reaches 321.3 K, the resulting \( h \) is 78.75. A sample graph showing the cooling of a wire for an arbitrary experiment can be found in the appendix.

This experiment is believed to have several weaknesses that could affect the reliability of the data. The anode and cathode in this experiment (the aluminum blocks) have an extremely large heat capacity relative to the wire. For this reason they act as heat sinks and draw heat from the wire. Our simplified model does not take this into account explicitly, thereby giving us inaccurately large values of \( h \). This is because the heat lost by conduction to the electrodes is lumped into the value of \( h \) because there is not a conduction term to account for it.

**Experiment 2:**

The goal of this experiment is to determine a value for the coefficient of convection that is more accurate than that obtained in the first experiment. We accomplish this by reducing the relative heat capacity of the electrodes such that conduction does not play such an integral role in the experiment. In this experiment we use a piece of nichrome wire of length .082 meters. The procedure is the same as in the first experiment except that the aluminum blocks (electrodes) are substituted with small clips that send the current through the wire. These clips absorb much less heat than the larger aluminum blocks. When the maximum temperature of the wire is 406.6 K, the resulting \( h \) is 50, and when the maximum temperature reaches 414.1 K, the resulting \( h \) is 63. Note that the lower values for \( h \) are predictable since there is less heat loss in general.

It is clear that conduction was reduced in this experiment but we cannot be certain that it is now negligible.

**Experiment 3:**

The goal of this experiment is to calculate \( h \) and be certain that conduction is playing a negligible role. To minimize the conduction term, and isolate convective heat loss, we
minimize the size of the electrodes and maximize the length of the wire, thus reducing the relative heat capacity of the electrodes. Replacing the aluminum blocks with metal clips in experiment 2 reduces the size of the electrodes. With the smaller electrodes there exists a length of wire such that conduction to the electrodes is relatively small and can be neglected. To be sure that we have effectively isolated convection, we heat a wire up to a designated temperature and perform the same calculations as in the previous experiments to determine \( h \). We then increase the length of the wire (reducing the relative heat capacity of the electrodes) and repeat the experiment. The value of \( h \) is compared to the previous trial. If the values from each trial are different, it can be concluded that conduction is still a significant factor. We repeat this experiment, each time increasing the length of the wire, until the coefficient of convection for trial (call it trial \( n \)) is negligibly different from the preceding trial (\( n-1 \)).

\[
\Delta h_n - h_{n-1} \approx 0
\]  

(38)

Once this condition is satisfied we can say that conduction is negligible and that the only significant heat lost is due to convection. We use the data gathered from each trial to calculate the value of the coefficient of convection for use in our mathematical model. The results of this experiment are as follows:

![H as a Function of Length](image)  

Figure 4
From the graph, no correlation can be made between lengthening the wire and converging on a value of $h$. Also, the $h$ values in experiment 3 are higher than those obtained in experiment 2. This is counter-intuitive because experiment 3 is meant to remove conduction, which would cause less heat to be lost from the wire and lower values of $h$. One possible source of error in this experiment is the thermocouple used to measure the temperature of the wire. The data recorded from this thermocouple oscillated by plus or minus 10 degrees even when the temperature of the wire was constant. This might be because one side of the thermocouple is exposed to the ambient air while the other side is in contact with the wire. To correct this, we plan on coating the thermocouple with some thermally conductive paste (possibly oil) to obtain more accurate readings.

Annotated Bibliography:

The following books and journals provide relevant information for our project. Their summaries can be found below.


This book derives a system of equations governing the electrostatic and thermal problems encountered during the joule heating of a cylinder. First, it uses Maxwell’s equations and Ohm’s law to derive an equation for the electrostatic potential. This is then combined with boundary conditions to form a system of equations representing the electrostatic problem. Next, it derives the heat equation as it applies to isotropic, homogeneous materials, which is what we will be working with. After a source term is added to the heat equation, it is also combined with boundary conditions to develop a system of equations representing the thermal problem. Since both systems depend on electric conductivity, they are combined into one system to be solved simultaneously. In addition, an in depth analysis of this system using scaled variables is done to obtain fewer non-dimensional parameters.


This article addresses that the electrical resistance of metals increases according to the elevation in temperature. Furthermore, it confirms our assumption that joule heating provides uniformly distributed cross-sectional temperature in the filament. This validates our supposition that the thermal model is only a function of time and distance along the filament.

This introductory college physics textbook provides us with some common equations to govern electrical and thermal conductivity. Specifically, it covered Ohm’s Law and the equation relating electrical resistivity to electrical resistance. It also discusses the linear relationship between resistivity and temperature over certain ranges of temperature.


This book provides us with information relevant to all of the modes of heat transfer occurring in our problem. Conduction occurs throughout the steel filament as well as from the ends of the filaments to the copper fingers. Convection occurs from the surface of the filaments to the surrounding air. Heat is also radiated from the surface of the filaments when they are heated. This book supplies equations that govern each of these modes of heat transfer as well as examples that closely resemble particular aspects of our problem.


The above two articles describe the non-local mathematical model of the joule heating of a cylinder. This model governs the temperature of a material, with temperature dependent electrical conductivity, when an electrical current is passed through it with a fixed voltage. These articles further discuss the situations in which a solution can be found for the model.
Appendix

Log(U) vs. time with $T_{max} = 307.6K$ and heat sinks