1 Background

Seeing the two-dimensional shadow of an object, one might ask whether it is possible to reconstruct the entire three-dimensional object. After some thought, you will find that it is possible for two different objects to cast the same shadow from certain angles, and so we realize that we cannot uniquely identify an object from its shadow without some extra information.

While the problem may appear hopeless, it is critical in fields like medical imaging, petroleum exploration and other types of remote sensing. Doctors have gleaned useful information from X-ray images for decades, even though they represent a two-dimensional shadow of a person’s body, because doctors also know something about comparative human anatomy and how X-ray images are supposed to look. Computer tomography images (i.e. CAT-scans and other technologies) are much more sophisticated and involve the synthesis of many shadows from different angles into a more comprehensive three dimensional model of our internal organs.

In this problem, we will focus on what is called the forward problem. Given a three-dimensional object, you will compute it’s shadow on different planes.

2 The problem

You will compute the shadows of a helical space curve

\[
\mathbf{r}(t) = \begin{bmatrix} e^{-t} \cos(4\pi t) \\ e^{-t} \sin(4\pi t) \\ t \end{bmatrix} \quad 0 \leq t \leq 5.
\]

onto the projection planes:

1. [Easy] \( z = -1 \),
2. [Easy] \( x = 2 \),
3. [Challenging] \( x + y + 10z = -10 \),
4. [Challenging] \( x + y + z = -2 \).

You must find equations for these curves on the planes, and plot the shadows on the projection plane. You may assume that the light source casting the shadow is infinitely far away, that the helix is between the light source and the plane, and that the light is traveling in a direction normal to the plane. You may have to make some additional assumptions to generate the two-dimensional equations and plots. Be sure to explain what assumptions you must make.