Final Exam
22 May 2002

“... I never went to Cambridge as my brothers did. I had the chance, but I refused it. I wanted to get out into the world. I’ve always regretted it. I think it would have saved me a lot of mistakes. You learn more quickly under the guidance of experienced teachers. You waste a lot of time going down blind alleys if you have no one to lead you.”

“You may be right. I don’t mind if I make mistakes. It may be that in one of the blind alleys I may find something to my purpose.”

from The Razor’s Edge by W. Somerset Maugham

Instructions: Show all work to receive full or partial credit. You may use a scientific calculator, Maple and one 8.5 X 11 inch page (front and back) of notes on this exam. Please hand in all Maple worksheets and your note page with your exam. All University rules and guidelines for student conduct are applicable.

1. [10 pts] Find the curvature of the path

   \[ \mathbf{r}(s) = (s, s^4, s^2) \]

   when \( s = 0 \).
2. [10 pts] For what value(s) of $x$ does the vector $\langle x, 0, 3 \rangle$ form an angle of $\frac{\pi}{6}$ with the vector $\langle \sqrt{6}, 1, 1 \rangle$?

3. [10 pts] Find an equation for the plane containing the line

$$x = 2y = 3(z - 1)$$

and the point $(1, 1, 1)$. 

4. [10 pts] Find $\nabla f$ and $\frac{\partial^2 f}{\partial x \partial y}$ if

$$f(x, y) = \frac{y}{4} (e^x + e^{-x})^2.$$ 

5. [10 pts] Find any local maxima, minima and saddle points, as well as the absolute maxima and minima of

$$z = x^2 - y^2$$

on the set $(x - 1)^2 + y^2 \leq 4.$
6. [10 pts] Find the mass and centroid of the object sketched below assuming the material has uniform density \( \rho = z(x^2 + y^2) \).
7. [12 pts] Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ over the path shown below if

$$\mathbf{F} = (e^x + y^2, 2xy + \cos(y)).$$
8. [13 pts] Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ over the path shown below if

$$\mathbf{F} = (-y, x).$$
9. [15 pts] Sketch the volume bounded by the surfaces \( z + x^2 = 1, \ z = 0, \ y = 1 \) and \( y = -1 \) and find the flux of

\[
\mathbf{F} = (e^y + x^3, \cos(x) + y, \sin(xy) + z)
\]

out the volume.

I hope you have enjoyed learning this material as much as I have enjoyed teaching it. With this course behind you, I hope you will have the chance to use and enjoy the concepts we have covered in future classes and your careers. Have a great summer.

\[-LFR\]