Math428: Problem Set #2

1. Write the differential equation

\[ y'' + (y')^2 / y - \sin(y) = 0, \quad y(0) = 1, \quad y'(0) = 1 \]

as a first order system of equations. Note: it will not necessarily be linear. Suggest a general class of \( n \)th-order nonlinear differential equations that can be converted to a first order system. Try to make your class as broad as possible.

2. Using Euler’s method, solve

\[ y' = -y, \quad y(0) = 1 \]

for \( 0 \leq t \leq 10 \) with step sizes of \( h = 0.5, 0.75, 1, 1.5, 2, 2.5 \). Plot the errors in the numerical computation as a function of time for the different step sizes all on the same graph. (Make sure your plot has a legend.) How does the computed solution compare to the exact solution for different values of \( h \)? Would a Taylor series method improve this computation at all?

3. Repeat problem #2 using the backward Euler’s method. Explain the difference in behavior by examining the numerical method as a difference equation.

4. Repeat problem #2 for the differential equation

\[ y' = -\left(1 + \frac{9}{10} \cos(t)\right)y, \quad y(0) = 1 \]

for \( 0 \leq t \leq 20 \). If you cannot find an exact solution, compute a reference solution by setting \( h \) to be very, very small. Explain the difference in behavior for different values of \( h \) by examining the varying decay rate of the exact system as a function of time.