Final Exam Problems
Top Sekrit!

Instructions: Show all work to receive full or partial credit. You may use a scientific calculator/graphing calculator. All University rules and guidelines for student conduct are applicable.

1. Find the angle between vectors $(0, 1, -2)$ and $(-4, 1, 1)$.

2. If

$$f(x, y) = e^x \cos(xy),$$

calculate $\nabla f$, $\frac{\partial f^2}{\partial x^2}$ and $\frac{\partial f^2}{\partial x \partial y}$.

3. Find the equation of the plane passing through the points $(0, 0, 1)$, $(3, 2, 1)$ and $(6, 0, 1)$.

4. What is the area of the triangle with vertices $(0, 1, 2)$, $(3, -1, 2)$, $(3, 3, -1)$?

5. Find the maximum and minimum value of

$$f(x, y) = x^2y + 3x^2y^2 + 2$$

for $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

6. Find the equation of the tangent plane to the surface

$$x^2 + \frac{y^2}{9} + z^2 = 1$$

at the point $(\sqrt{2}, 3, 1)$.

7. Calculate the directional derivative of

$$f(x, y) = e^x(x + y^2)$$

at $(0, 1)$ in the direction $3\hat{i} + 4\hat{j}$.

8. If

$$\vec{f}(x, y, z) = xy\hat{i} + e^{yz}\hat{j} + \ln(1 + x^2 + y^2 + z^2)\hat{k}$$

find $\nabla \cdot \vec{f}$ and $\nabla \times \vec{f}$.

9. Find the volume under the surface

$$f(x, y) = 4 - x^2 - y^2$$

over the region $R = \{(x, y)|x^2 + y^2 \leq 4\}$.
10. Find the centroid of the object sketched below assuming that it has uniform density.

![Graph showing an object with coordinates and dimensions for finding the centroid.]

11. Find the flux of

\[ \vec{f}(x, y, z) = x^2 \hat{i} + z \hat{j} + e^z \hat{k} \]

on the cube shown below.

![Diagram of a cube with vertices labeled for finding the flux through a surface.]

12. Find \( \int_C \vec{F} \cdot d\vec{x} \) where

\[ \vec{f}(x, y) = (\cos(x) + y^2) \hat{i} + 2xy \hat{j} \]

and \( C \) is the path shown below leading from \((1, -5)\) to \((-1, -5)\).

![Diagram of a path in a plane leading from one point to another for line integral calculation.]

13. Find the mass and centroid of the object sketched below assuming the material has uniform density \( \rho = 1 \).

![Graph showing an object with coordinates and dimensions for finding the mass and centroid.]

14. Calculate

\[ \iiint_R f(x, y, z) dxdydz \]

where

\[ f(x, y, z) = xyz \]

and \( R \) is the region bounded by the \( xy \)-plane, \( xz \)-plane, \( yz \)-plane, the plane \( y = 2 \) and the plane

\[ x + z = 1. \]

15. Calculate \( \int_C \vec{f} \cdot d\vec{x} \) where

\[ \vec{f}(x, y) = (\cos(\pi x) + y)\hat{i} + \left( x + \frac{1}{1 + y} \right)\hat{j} \]

over the path \( C \) shown below.

16. Calculate \( \int_C \vec{f} \cdot d\vec{x} \) where

\[ \vec{f}(x, y) = xy\hat{i} + (x^2 + z)\hat{j} + x\hat{k} \]

where \( C \) is the line segment connecting \((0, 0, 0)\) to \((1, 2, 3)\).
17. Find

\[ \int_C \mathbf{f} \cdot d\mathbf{x} \]

where

\[ \mathbf{f}(x, y, z) = (x + y)\mathbf{i} + (x + z)\mathbf{j} + 6x\mathbf{k} \]

over the path \( C \).

18. Calculate the center of mass of this 3/4 section of a washer.

19. Calculate

\[ \iint_R zdS \]

where \( R \) is the upper surface of a sphere of radius 2 centered at the origin.

20. Find a potential for the following vector field.

\[ \mathbf{F} = \sin y\mathbf{i} + (x \cos y + y^2)\mathbf{j} + e^{3z}\mathbf{k} \]