Problem 1: Movin’ the Couch

- Jack wants to move his couch so he has to walk as little as possible to reach the “essential” parts of his apartment.
The Layout...

Let’s assume there are no walls, doors, or stairs to complicate the problem.

We’ll place the couch at an arbitrary point \((x,y)\) and minimize the sum of the distances between the couch and the vertices of the triangle.
What to do?

\[ D = \sqrt{x^2 + y^2} + \sqrt{x^2 - 2xa + a^2 + y^2} + \sqrt{x^2 - 2xb + b^2 + y^2 - 2yc + c^2} \]

The partial derivative with respect to x:

\[ \frac{x}{\sqrt{x^2 + y^2}} + \frac{2x - 2a}{2\sqrt{x^2 - 2xa + a^2 + y^2}} + \frac{2x - 2b}{2\sqrt{x^2 - 2xb + b^2 + y^2 - 2yc + c^2}} \]

The partial derivative with respect to y:

\[ \frac{y}{\sqrt{x^2 + y^2}} + \frac{y}{\sqrt{x^2 - 2xa + a^2 + y^2}} + \frac{2y - 2c}{2\sqrt{x^2 - 2xb + b^2 + y^2 - 2yc + c^2}} \]

These equations were too complex for even MAPLE to solve.
Some Hairy Math…

• We tried to solve a simplified equation by squaring the three components of D.

  • This method gave partial derivatives that were easy to solve…but unfortunately did not give the right answer.

  • For example, the triangle formed by the points (0,0), (5,0), (7,5) would give $D_x = 6x-24$, $D_y = 6y-10$ putting the couch at $x = 4$, and $y = 5/3$. This gives a total distance equal to 10.7. Choosing another point to put the couch (5,0), gives a distance equal to 10.38.
Minimizing the square root of $x$ is the same as minimizing $x$ only when $x$ is greater than or equal to one. Maybe our answer lies on points where $x$ and $y$ are not greater than or equal to one?

Instead, we took the natural log of both sides and minimized the natural log of $D$. The partial derivatives of this function, $d$, looked nearly identical as the original ones, but because $\ln(x)^n = n\ln(x)$ we were able to get rid of the square roots.

\[
\begin{align*}
    dx &= \frac{x}{x^2 + y^2} + \frac{2x - 2a}{2(x^2 - 2xa + a^2 + y^2)} + \frac{2x - 2b}{2(x^2 - 2xb + b^2 + y^2 - 2yc + c^2)} \\
    dy &= \frac{y}{x^2 + y^2} + \frac{y}{x^2 - 2xa + a^2 + y^2} + \frac{2y - 2c}{2(x^2 - 2xb + b^2 + y^2 - 2yc + c^2)}
\end{align*}
\]
Next, we put in some values representing possible locations for the DVD player, the bathroom and the fridge, and solved the partial equations for $x$ and $y$. For the triangle used earlier, the minimum value was found at $x \approx 4.76$, $y \approx 0.35$. 

Using MAPLE
In Conclusion…

• Given the three coordinates that represent Jack’s living room and a few minutes, we could now tell him where Diane needs to move his couch.

You want it where!?!
Problem 2: Feedin’ the Cows

- **Goal**: To maximize the volume of a feed trough to be built from a 6’x10’ sheet of metal.
- The metal body will be set into two wooden end pieces.
The Setup

• How big should the folds be?
• At what angle should the metal be bent?
The Formulas

Cross-Sectional Area of Trough:
\[ \frac{1}{2} r \sin(t) \left( 12 - 4 r + 2 r \cos(t) \right) \]

Partial Derivative with Respect to r:
\[ \frac{1}{2} \sin(t) \left( 12 - 4 r + 2 r \cos(t) \right) + \frac{1}{2} r \sin(t) \left( -4 + 2 \cos(t) \right) \]

Partial Derivative with Respect to t (θ):
\[ \frac{1}{2} r \cos(t) \left( 12 - 4 r + 2 r \cos(t) \right) - r^2 \sin(t)^2 \]
Solving the Problem

• Set $A_r$ to zero, and solve for $r$:
  $$r = \frac{-3}{-2+\cos(t)}$$

• Substituting the above for $r$ in $A_t$ and solving for $t$: $t = \frac{\pi}{3}, \ -\frac{\pi}{3}$

• For $t = \frac{\pi}{3}$, $r = 2$

The finished trough holds approximately 52 cubic feet of cow feed