NOTE: SHOW ALL YOUR WORK. ALL PROBLEMS ARE EQUALLY WEIGHTED.
(You may use MATLAB to perform your computations, whenever it is convenient to you.)

1. Given the matrix
\[
A = \begin{bmatrix}
1 & -2 & 1 & 1 & 2 \\
-1 & 3 & 0 & 2 & -2 \\
0 & 1 & 1 & 3 & 4 \\
1 & 2 & 5 & 13 & 5
\end{bmatrix},
\]
find the following:
(a) the rank of \( A \), (b) a basis for the null space of \( A \), (c) a basis for the row space of \( A \) and (d) a basis for the column space of \( A \).

2. Let \( v \) be an inner product space. Suppose that \( W \) be a subspace of \( V \) and \( w \neq 0 \) be a fixed vector in \( W \). Show that for any \( v \in V \),
\[
\alpha := \frac{(v, w)}{||w||^2}
\]
is the unique scalar such that
\[
w^\perp := v - \alpha w
\]
is orthogonal to \( w \).

3. Let \( C[-\pi, \pi] \) be the inner product space of real-valued continuous functions on the interval \([−\pi, \pi]\) with inner product defined by
\[
(f, g) := \int_{-\pi}^{\pi} f(t)g(t)dt, \quad f, g \in C[-\pi, \pi].
\]
(a) Use the Gram-Schmidt process to obtain an orthonormal basis for the subspace \( W = \text{span}\{1, \cos t, \sin t\} \) of the inner product space.

4. Let \( W \) be the subspace of continuous functions on \([−\pi, \pi]\) in Problem 3. Write the vector \( f = t - 1 \) as
\[
f = w + w^\perp
\]
with \( w \in W \) and \( w^\perp \in W^\perp \).

5. Let \( W \) be the plane : \( 3x + 2y - z = 0 \in \mathbb{R}^3 \).
(a) Show that
\[
W^\perp := \{ v \in \mathbb{R}^3 \mid v \perp W\}
\]
is a subspace of \( \mathbb{R}^3 \).
(b) Find a basis for the subspace \( W^\perp \).