MATH 508 Challenge problem #6
Turn in no later than Apr. 5 for credit

Suppose you have two coins, one of radius $R$ and another of radius $nR$ for $n \geq 1$. Let the larger coin be fixed with its center at the origin, and rotate the smaller coin around it (without slipping). If $n$ is an integer, a fixed point on the rim of the smaller coin (say, a dab of ink) traces out a closed curve called an epicycloid.

Use Maple to make a plot of the epicycloid when $R = 1$ and $n = 5$. Then, use (without proof) the formula

$$\int_{\Gamma} z\,dz = 2i\text{(area of } D),$$

where $\Gamma$ is a simple closed curve enclosing domain $D$, to find the area of the domain enclosed by a general epicycloid. (You may use Maple to assist with the calculation, if you like.)