MATH201
Summer College 2004

Exam 4

Name: Key /50

Instructions:

1. Do not start until instructed to do so.
2. You may use a calculator and one 3” x 5” card (front and back) with notes, but nothing else.
3. The work you turn in must be your own.
4. SHOW ALL WORK.
Questions 1 - 2: An insurance company checks police records on 582 accidents selected at random and notes that teenagers were at the wheel in 91 of them. A politician urging tighter restrictions on drivers’ licenses issued to teens says, “In one of every 5 auto accidents a teenager is behind the wheel.”

1. **4 points** Construct a 98% confidence interval for the proportion of all auto accidents that involve teenage drivers.

\[
\hat{p} = \frac{\bar{x}}{n} = \frac{91}{582} \approx 0.1571
\]

\[
\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.1571 \pm 2.33 \sqrt{\frac{0.1571(1-0.1571)}{582}}
\]

\[
(0.121, 0.191)
\]

2. **3 points** Do the data support or contradict the politician’s statement? Explain.

The data contradict the politician’s statement: \( \frac{1}{5} = 0.20 \) is not in the interval of plausible values. It seems the proportion of teenage accidents is less than 1 out of 5.

3. **3 points** During routine screening, a doctor notices that 22% of his adult patients show higher than normal levels of glucose in their blood—a possible warning signal for diabetes. Hearing this, some medical researchers decide to conduct a large-scale study, hoping to estimate the proportion to within 4% with 90% confidence. How many randomly selected adults must they test?

\[
n = \frac{Z_{\alpha/2} \hat{p}(1-\hat{p})}{\beta^2} = \frac{1.645^2(0.22)(0.78)}{0.04^2} \approx 290.2
\]

4. **3 points** Find the value of t so that the area to its right under the t-distribution is 0.975. Use df = 15.

\[
t = -2.131
\]

5. **4 points** A city wants to build a new public parking garage in the central business district and plans to pay for the structure through parking fees. In order to estimate the average daily income from parking fees, the planning committee decides to collect data from an existing garage nearby. How many day’s worth of data must they collect to have 99% confidence of estimating the true mean to within $3? Previous data show that the standard deviation of daily parking fees is around $15.

\[
n = \left( \frac{2.575 \times 3}{15} \right)^2 = \left( \frac{9}{3} \right)^2 = 16.576
\]

\[
2.575
\]
6. **3 points** Match the correct symbol with each description below. Symbols may be used more than once or not at all.

- $\alpha$ - the significance level of a hypothesis test
- $\beta$ - the multiplier used in a confidence interval formula that is related to the confidence level
- $\alpha/2$ - the probability of making a Type II error
- $\beta/2$ - 1 - confidence level
- $z_{\alpha/2}$ - the limit on the probability of making a Type I error

**Questions 7-8:** Psychology experiments sometimes involve testing the ability of rats to navigate mazes. The mazes are classified according to difficulty, as measured by the mean length of time it takes rats to find the food at the end. One researcher needs a maze that will take rats an average of about one minute to solve. He tests one maze on 20 rats and obtains their times in seconds. The data are summarized below. Assume that completion times are normally distributed.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>TrMean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>20</td>
<td>50.13</td>
<td>49.95</td>
<td>49.97</td>
<td>5.90</td>
<td>2.21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>33.90</td>
<td>69.20</td>
<td>41.75</td>
<td>57.28</td>
</tr>
</tbody>
</table>

7. **1 point** Give the point estimate.

$$\bar{x} = 50.13$$

8. **5 points** Construct and interpret a 95% confidence interval for the average completion time for all rats.

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$50.13 \pm 2.093 \frac{9.90}{\sqrt{20}} \rightarrow (45.50, 54.76)$$

We are 95% confident that the avg. completion time for all rats is between 45.5 and 54.76 seconds... a bit too easy!

9. **3 points** In our class survey, we collected data on the number of CDs owned by each student. Summaries are shown below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>TrMean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDs</td>
<td>22</td>
<td>77.6</td>
<td>30.0</td>
<td>47.6</td>
<td>161.9</td>
<td>34.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDs</td>
<td>3.0</td>
<td>753.0</td>
<td>6.5</td>
<td>66.5</td>
</tr>
</tbody>
</table>
if you wanted an interval estimate of the number of CDs owned by the average high school senior, which confidence interval tool would be appropriate?

\[ \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \]

\[ \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \]

\[ \bar{x} \pm t_{\alpha} \frac{s}{\sqrt{n}} \]

\[ \bar{x} \pm \frac{t_{\alpha}}{\sqrt{n}} \]

E. Sign interval

Questions 10 – 11: The manufacturer of a metal stand for home TV sets must be sure that its product will not fail under the weight of the TV. Since some larger sets weigh nearly 300 pounds, the company’s safety inspectors have set a standard of ensuring that the stands can support an average of over 500 pounds. Their inspectors regularly subject a random sample of the stands to increasing weight until they fail. They test the hypotheses \( H_0: \mu \leq 500 \) versus \( H_a: \mu > 500 \) where \( \mu \) = average weight limit for all TV stands.

10. 3 points Explain what a Type I error would be in terms of this problem. **Reject** \( H_0 \) when \( H_0 \) is true

Concluding that the avg. wt. limit for all TV stands is
over 500 lbs. when in fact it isn’t.

11. 3 points Explain what a Type II error would be in terms of this problem. **Do not reject** \( H_0 \) when \( H_0 \) is true

Failing to conclude that the avg. wt. limit for all TV stands is
over 500 lbs. when in fact it is.

12. 3 points The state police in DE have embarked on a "Click It or Ticket" campaign with the goal of achieving compliance with the law requiring all drivers to wear seatbelts. Suppose the state police want to check to see if compliance is more than 80% and to do that they randomly sample drivers and check to see if they are wearing their seatbelt. Set up the null and alternative hypotheses for this study. Be sure to refer specifically to the population characteristic (parameter) under study.

\( H_0: \) proportion of drivers in DE who wear seatbelt \( \leq .80 \)

\( H_a: \) proportion of drivers in DE who wear seatbelt \( > .80 \)
Questions 13 – 14: Suppose you want to get an idea of what people think about the importance of statistics. To do this you design a survey with this question: “Given the fact that those who understand statistics are smarter and better looking than those who don’t, don’t you think it is important to take a course in statistics?” You survey 2000 people and find that 1800 of them answered “yes.”

13. 3 points Find the margin of error for this survey if you use 90% confidence.

\[
\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.9(1-0.9)}{2000}}
\]

14. 3 points Explain how this margin of error might be misleading and how you would improve the study.

It may underestimate the true error! The question is biased toward “yes” and the MOE doesn’t take that into account. The true proportion of people who think taking a stat course is important is probably much lower than the MOE indicates. I would change the question to “Do you think taking a stat course is important?”

15. 2 points An analysis of the salaries of a random sample of 288 Nevada teachers produced a 90% confidence interval of ($38,944, 42,893). Which interpretation below is correct? (circle only one)

a. 90% of the teachers studied earned between $38,944 and $42,893.

b. We’re 90% confident that a Nevada teacher will earn between $38,944 and $42,893.

c. We’re 90% confident that the average salary for the teachers in this study is between $38,944 and $42,893.

d. If another random sample of 288 Nevada teachers is taken, there is a 90% chance that their average salary will be between $38,944 and $42,893.

e. We’re 90% confident that the interval captures the average salary for all Nevada teachers.

16. 2 points A confidence interval is reported for some survey research. Which of the following is accounted for by the margin of error? (circle only one)

a. the fact that sample results vary from one randomly selected sample to the next

b. problems with the wording of questions

c. a sample that does not represent the population of interest

d. bias introduced by some people not responding

17. 2 points The t-distribution

a. is centered at 0 like the z-distribution.

b. is used to account for more uncertainty when \( s \) is substituted for \( \sigma \).

c. is “shorter” with more area under the tails than the z-distribution.

d. is based on degrees of freedom \( n - 1 \).

e. has all of the above characteristics.