EXAMPLES OF NON-HOMOGENEOUS SECOND ORDER LINEAR EQUATIONS

1. \[ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x \]

2. \[ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 3x^2 \]
3. \[ y'' - 16y = \sin(2x), \]

with the initial conditions

\[
\begin{align*}
y(1) &= 1, \\
y'(1) &= 0,
\end{align*}
\]

given that the general solution of the homogeneous problem is \[ y_h(x) = c_1e^{-4x} + c_2e^{4x}. \]
4. \[ y'' + 16y = \sin(8x), \]
with the initial conditions
\[ \begin{cases} y(1) = 1 \\ y'(1) = 0 \end{cases} \]

5. \[ z'' - 2z' + z = e^{-x}, \quad \text{with} \quad z(0) = 0, \quad z'(0) = 1, \]
given that the general solution to the homogeneous problem is \( z_h(x) = c_1 e^x + c_2 xe^x. \)
6. \[ \ddot{x} + 10 \dot{x} + 24x = t + e^{2t}. \]

7. \[ \frac{d^2x}{dt^2} + 8 \frac{dx}{xt} + 25x = \cos(2t), \] with \( x(0) = 0, \) \( \left( \frac{dx}{dt} \right)(0) = \frac{1}{2}. \)

given that the general solution of the homogeneous equation is \( x_h(t) = e^{-4t} [c_1 \cos(3t) + c_2 \sin(3t)]. \)

What result do you get when you replace the right hand side with \( \cos(3t)? \)