Examples of Partial Derivatives

The following are routine examples of computing partial derivatives of functions of several variables. You have some assigned from your text book. These we will go through in class.

A word on notation. The notation for partial derivatives that we discussed in class is, for example,

\[ \frac{\partial f}{\partial x} \text{ or } \frac{\partial f}{\partial y}. \]

As we explained in class, partial derivatives, as is the case with ordinary derivatives, are defined in terms of a limit at a particular point in the domain of \( f \). If we look at the derived function \( (x, y) \mapsto \frac{\partial f}{\partial x}(x, y) \), then we can define the partial derivatives of this derived function. This leads to the second partial derivatives of a function. In the case of two variables, there are three,

\[ \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \text{ and } \frac{\partial^2 f}{\partial y^2}. \]

You will, however, need to get used to an alternative notation that is very commonly used, namely the use of literal subscripts to denote partial differentiation with respect to the named variable. Thus \( f_x \) stands for the first partial derivative of \( f \) with respect to its variable \( x \), while \( f_{xy} \) stands for the “mixed” partial, first with respect to \( y \) and then with respect to \( x \) i.e.,

\[ \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = f_{xy}. \]

This notation is most often interpreted as involving the derived function and not the partial derivative of \( f \) at some particular point.
Examples:
In each case we will compute some of the derivatives, $f_x, f_y, f_z, f_{xy}, f_{yx},$
$f_{xx}, f_{yy}, \ldots$

1. 
   
   \[ f(x, y, z) = x^2 e^{yz}. \]

2. 
   
   \[ f(x, y) = \sqrt{x^2 + y^2}. \]
3.

\[ f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}. \]
4.

\[ f(x, y) = \frac{1}{\sqrt{y}} e^{-(x-a)^2/4y}. \]