Math 512: Computational challenge for Fall 2004

Void fractions

Consider a one-dimensional array of \( n \) empty spaces to be filled with “dominoes,” each of which occupy two spaces. To be admissible, the array must be filled such that no two consecutive spaces are empty. For example, if \( n = 6 \), three possible configurations are

\[
\begin{array}{ccccccc}
X & X & X & X & X & X & X \\
\end{array}
\]

There are other possibilities as well.

The void fraction of a configuration of dominoes is \( k/n \) where \( k \) is the number of vacant spaces. The void fractions for the examples above are \( 1/3 \), \( 1/3 \) and 0, respectively. The mean void fraction, \( f(n) \), is defined to be

\[
f(n) = \frac{\sum \text{all admissible configurations} \ k/n}{\text{total number of admissible configurations}}
\]

The assignment is to calculate or estimate \( f(n) \) for \( 2 \leq n \leq 50 \). You may use any computer language or software package you like. You should hand in your source code, your results and an explanation of your algorithm, methodology and tools.