Updates

1. Writing assignment outlines are due on Thursday, April 4.

Homework Set 6


The Stefan Problem

1. (12 points) Consider the one-phase Stefan problem where the condition at $\tilde{x} = 0$ is a flux condition:

$$-k \frac{\partial \tilde{T}}{\partial \tilde{x}}(0, \tilde{t}) = Q,$$

where $Q$ is a constant. Show that this problem may be scaled in such a way so that no dimensionless parameters appear. Why does this happen?

Advanced ODEs

2. (10 points) Write the solution to

$$u'' = f(x), \quad 0 < x < 1; \quad u'(0) = u'(1) = 0,$$

in terms of a (regular or modified) Green’s function. Be sure to state any restrictions on $f$ and how they affect any arbitrariness in your solution.

3. (18 points) Write the solution to

$$(xu')' = f(x), \quad 1 < x < e; \quad u(1) = 0, \quad eu'(e) = u(e),$$

in terms of a (regular or modified) Green’s function. You should find that part of the Green’s function looks like

$$\frac{-x(\log x - 2) \log \xi + e \log(\xi/x)}{e - 2}.$$

Be sure to state any restrictions on $f$ and how they affect any arbitrariness in your solution.