Updates

1. The topic for your writing assignment is due Thursday, Feb. 28.

Homework Set 3

Read sections G1.2, G1.6, G2.6, G4.1, L7.4, W1–W3, W8, and W9.

Classification of Second-Order PDEs

1. (8 points) For the equation in Guenther and Lee, page 46, exercise 5(d), classify it and transform it into canonical form. Discuss any singularities that occur in the transformed equations. Do NOT restrict the domain.

2. Suppose that we wish to impose Cauchy data

\[ \phi = \phi(\eta), \quad \frac{\partial \phi}{\partial \xi} = h(\eta) \quad (3.1) \]

along on a characteristic.

(a) (12 points) Show that we may do so only if

\[ G[\xi, \eta] h' + F[\eta] \phi'' + H[\xi] h + H[\eta] \phi' + F \phi + G = 0, \quad (3.2a) \]

\[ H[\xi] = A \frac{\partial^2 \xi}{\partial x^2} + B \frac{\partial^2 \xi}{\partial x \partial y} + C \frac{\partial^2 \xi}{\partial y^2} + D \frac{\partial \xi}{\partial x} + E \frac{\partial \xi}{\partial y}. \quad (3.2b) \]

(b) (3 points) Explain why if (3.2) is satisfied, there are an infinite number of solutions to the original PDE.

The Wave Equation

3. (a) (6 points) Guenther and Lee, page 5, exercise 1. Let the free end of the chain correspond to \( x = 0 \).

(b) (3 points) Simplify your answer to (a) in the case that the density of the chain is constant.
4. (8 points) Use the general solution to solve the following system:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}; \quad x > 0, \quad t > 0,$$

(3.3a)

$$\psi(x, 0) = \frac{\partial \psi}{\partial t}(x, 0) = 0, \quad \frac{\partial \psi}{\partial x}(0, t) = h(t).$$

(3.3b)

Discuss the possibility of discontinuities in your solution, and explain your solution in terms of signal speed, range of influence, etc.