Updates

1. The final exam will be held on Monday, Dec. 10 from 3:30–6:30 pm.

Homework Set 8

Read sections 7.2–7.4.1, 8.1, 8.4.

Shock Waves

1. Consider the following equation:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad u(x, 0) = x - 2n, \quad x \in (2n - 1, 2n + 1). \tag{8.1}
\]

(a) (10 points) Construct the solution. Show that the shock positions are stationary, and the shock strength decays like \( t^{-1} \) as \( t \to \infty \).

(b) (4 points) Draw the characteristic diagram, indicating the position of the shocks.

(c) (4 points) Sketch \( u \) for various values of \( t \).

The Wave Equation

2. The telegraph equation has the form

\[
\frac{\partial^2 u}{\partial x^2} = LC \frac{\partial^2 u}{\partial t^2} + (RC + GL) \frac{\partial u}{\partial t} + RG u, \tag{8.2}
\]

where all the parameters are positive.

(a) (5 points) Let \( u = v(x, t)e^{\alpha t} \) and choose \( \alpha \) such that the differential equation for \( v \) has no \( \partial v/\partial t \) term.

(b) (3 points) Show (by calculation, NOT direct substitution) that if \( RC = GL \), your answer to (a) reduces to the standard wave equation for \( v \).

(c) (4 points) Show that a signal arriving at \( x = b \) is simply a damped version of a signal sent at \( x = 0 \).
The Navier-Stokes Equations

3. Consider the flow and density given by

\[ x = a, \quad y = b + c \cos \omega t, \quad \rho = \frac{\rho_0}{1 + y^2}. \]  

(8.3)

(a) (2 points) Show that the flow in (8.3) is not steady. Describe it.

(b) (3 points) Calculate \( \frac{D\rho}{Dt} \).

Conservation of Momentum

4. (5 points) Suppose that a fluid is in steady motion past a bounded obstacle, and further suppose that \( f \equiv 0 \). Let \( K \) be the force acting on the obstacle. Use the linear momentum transfer equation:

\[
\frac{d}{dt} \int_R \rho \mathbf{v} \, dV = \int_R \rho f \, dV + \int_{\partial R} [t - \rho \mathbf{v} \cdot (\mathbf{v} \cdot \mathbf{n})] \, dA, 
\]  

(8.4)

to deduce that

\[
K = \int_S [t - \rho \mathbf{v} \cdot (\mathbf{v} \cdot \mathbf{n})] \, dA, 
\]  

(8.5)

where \( S \) is any (imaginary) surface enclosing the obstacle. (In practice \( S \) is chosen to simplify calculations). Be sure to explain why the sign of \( K \) is as indicated.

(Hint: What is \( R \)?)