Simple Step Forcing

Graphed below is the forcing function \( f(t) = 1 - 2u_\pi(t) \) for \( t \in [0, 2\pi] \).

Here is a graph of the solution (for the same interval) of

\[ \ddot{y} + y = f(t), \quad y(0) = \dot{y}(0) = 0. \]
Here is a graph of $\dot{y}$ vs. $t$ for the same range. Again, there are no discontinuities, but we see that the derivative of $\dot{y}$ (namely $\ddot{y}$) is discontinuous.

Here is a graph of $\ddot{y}$ vs. $t$ for the same range. Since $y$ and $\dot{y}$ are continuous, but the forcing is not, $\ddot{y}$ must be discontinuous.