Phase Plane: Real Eigenvalues

For the system

$$\dot{x} = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} x,$$

the solution is

$$x = c_1 e^{-5t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$  

Since we have one positive and one negative eigenvalue, we have a saddle point, as shown below. Note the straight lines corresponding to the eigenvectors.
For the system

\[
\begin{pmatrix}
-3 & 1 \\
1 & -3
\end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},
\]

the solution is

\[
\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^{-4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
\]

Since we have two negative eigenvalues, we have a stable node, as shown below.

Phase plane of (2).