S7.1-4. Sol: $\mathbb{P}(b^2 - 4 \geq 0) = \mathbb{P}(b > 2 \,\text{or}\, b < -2) = 1/3.$

S7.1-6. Sol: Let X be uniformly distributed in $(0, L)$. The desired probability is equivalent to

$$\mathbb{P}(X \leq L/3 \,\text{or}\, X \geq 2L/3) = \mathbb{P}(X \leq L/3) + \mathbb{P}(X \geq 2L/3) = 2/3.$$

S7.2-6. Sol: (a).

$$\mathbb{P}(X > 35.5) \;=\; \mathbb{P}\left(\frac{X - 35.5}{4.8} > \frac{35.5 - 35.5}{4.8}\right) = 1 - \Phi(0) = 0.5.$$

(b.) The desired probability is given by

$$\begin{aligned}
\mathbb{P}(30 < X < 40) &= \mathbb{P}\left(\frac{30 - 35.5}{4.8} < Z < \frac{40 - 35.5}{4.8}\right) \\
&= \Phi(0.94) - \Phi(-1.15) \\
&= \Phi(0.94) + \Phi(1.15) - 1 \\
&= 0.8264 + 0.8749 - 1 = 0.701.
\end{aligned}$$

S7.2-10. Sol 1: We must find x so that $\mathbb{P}(110-x < X < 110+x) = 0.50$, or, equivalently,

$$\mathbb{P}\left(\frac{-x}{20} < \frac{X-110}{20} < \frac{x}{20}\right) = 0.50.$$

Therefore, we must find the value of x which satisfies $\mathbb{P}(-x/20 < Z < x/20) = 0.50$ or $\Phi(x/20) - \Phi(-x/20) = 0.50$. Since $\Phi(-x/20) = 1 - \Phi(x/20)$, x satisfies $2\Phi(x/20) = 1.50$ or $\Phi(x/20) = 0.75$. Using Table 1 of the appendix, we get $x/20 = 0.67$ or $x = 13.4$ So the desired interval is $(110 - 13.4, 110 + 13.4) = (96.6, 123.4)$.

Sol 2: We must find x so that $\mathbb{P}(X \leq 110 - x) = 0.25$, or, equivalently, $\mathbb{P}(Z \leq -x/20) = 0.25$. Using table, we get $-x/20 \approx -0.67$ and $x = 13.4$.

S7.3-2. Sol: Let Q be the median of an exponential random variable with rate λ. Then $\mathbb{P}(X > Q) = 1/2$; thus $\int_0^Q \lambda e^{-\lambda x} dx = 1/2$, or $Q = \lambda^{-1} \ln 2$.

S7.3-6. Sol: We have

$$\begin{aligned}\mathbb{P}(|X-\mathbb{E}\,X|\geq 2\sigma_X) &= \mathbb{P}(|X-\lambda^{-1}|\geq 2\lambda^{-1})\\ &= \mathbb{P}(X-\lambda^{-1}\geq 2\lambda^{-1})+\mathbb{P}(X-\lambda^{-1}\leq -2\lambda^{-1})\\ &= \mathbb{P}(X\geq 3\lambda^{-1})+\mathbb{P}(X\leq -\lambda^{-1}) = e^{-3}+0\end{aligned}$$

R7-2. Sol: Let X be the weight of a randomly selected women from this community. The desired quantity is

$$\begin{aligned}\mathbb{P}(X>170|X>140) &= \frac{P(X>170)}{P(X>140)}\\ &= \frac{\mathbb{P}(Z>2)}{P(Z>0.5)} = \frac{1-\Phi(2)}{1-\Phi(0.5)} = 0.074.\end{aligned}$$

R7-14. Sol: We want to find the smallest a for which $P(X\leq a)\geq 0.90$. This implies $\mathbb{P}(Z\leq \frac{a-175}{22})\geq 0.90$. Using Table 1 of the appendix, we see that $(a-175)/22 = 1.29$ or $a = 203.38$.