S6.1-2. Sol: (a). $f(x) = \begin{cases} 32/x^3 & x \geq 4 \\ 0 & \text{otherwise.} \end{cases}$
(c). $\mathbb{P}(X \leq 5) = 9/25$; $\mathbb{P}(X \geq 6) = 4/9$;
$\mathbb{P}(5 \leq X \leq 7) = \int_5^7 f(x)dx = (16/25) - (16/49)$; $\mathbb{P}(1 \leq X < 3.5) = 0$.
S6.1-8. Sol: (a). $k = -1/2$ since $\int_{-1}^0 f(x)dx = 1$; (b). $\mathbb{P}(X \geq -1/2) = \int_{-1/2}^0 f(x)dx = 3/16$.
S6.2-2. Sol: The set of possible values of Y is $(0, \infty)$. For $y > 0$, $F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(e^X \leq y) = \mathbb{P}(X \leq \ln y) = F(\ln y)$. Therefore,
$f_Y(y) = F'_Y(y) = \begin{cases} f(\ln y)/y, & y > 0 \\ 0 & \text{otherwise.} \end{cases}$
S6.2-8. Sol 1: The set of possible values of Y is $(0, 1]$ For $0 < y < 1$, we have

$$\begin{aligned} F_Y(y) &= \mathbb{P}(Y \leq y) = \mathbb{P}(X \leq y \text{ or } X \geq 1/y) \\ &= \mathbb{P}(X \leq y) + \mathbb{P}(X \geq 1/y) \\ &= \int_0^y e^{-x}dx + \int_{1/y}^\infty e^{-x}dx \\ &= 1 - e^{-y} + e^{-1/y}. \end{aligned}$$

$f_Y(y) = F'_Y(y) = \begin{cases} e^{-y} + e^{-1/y}/y^2, & 0 < y < 1 \\ 0 & \text{o.w.} \end{cases}$
Sol 2: We can use conditioning for $0 \leq y \leq 1$: $F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(Y \leq y | X \leq 1)\mathbb{P}(X \leq 1) + \mathbb{P}(Y \leq y | X > 1)\mathbb{P}(X > 1) = \mathbb{P}(X \leq y | X \leq 1)\mathbb{P}(X \leq 1) + \mathbb{P}(X \geq 1/y | X > 1)\mathbb{P}(X > 1) = \mathbb{P}(X \leq y) + \mathbb{P}(X \geq 1/y)$.

S6.3-4. Sol: $\mathbb{E}\, e^X = \int_0^\infty e^x f(x)dx = 3/2$.
S6.3-6. Sol: Let f be the p.d.f of Y. Then

$$f(y) = F'(y) = \begin{cases} (k/A)e^{-k(\alpha-y)/A} & -\infty < y < \alpha \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, $\mathbb{E}\,(Y) = \int_{-\infty}^{\alpha} \frac{k}{A} y e^{-k(\alpha-y)/A} dy = \alpha - \frac{A}{k}$.
R6-4. Sol: We have

$$\begin{aligned} \mathbb{P}(-2 < X < 1) &= \int_{-2}^{1} e^{-|x|}/2dx = \int_{-2}^{0} e^{x}/2dx + \int_{0}^{1} e^{-x}/2dx \\ &= 1 - (2e)^{-1} - (2e^2)^{-1} \end{aligned}$$

R6-6. Sol: The set of possible values of Y is $[e, e^2]$. For $e < y < e^2$, $F_Y(y) = \mathbb{P}(Y \le y) = \mathbb{P}(e^X \le y) = \mathbb{P}(X \le \ln y) = \int_1^{\ln y} 4x^3/15dx = ((\ln y)^4 - 1)/15$. Thus

$$f_Y(y) = F_Y'(y) = \begin{cases} 4(\ln y)^3/(15y) & e < y < e^2 \\ 0 & \text{otherwise.} \end{cases}$$

Applying the same procedure to Z and W, we obtain $f_Z(z) = F_Z'(z) = \begin{cases} 2z/15 & 1 < z < 4 \\ 0 & \text{otherwise.} \end{cases}$ and

$$f_W(w) = F_W'(w) = \begin{cases} \frac{2(1+\sqrt{w})^3}{15\sqrt{w}} & 0 < w < 1 \\ 0 & \text{otherwise.} \end{cases}$$