Homework Solutions

S1.2-10: (a). $(E \cup F)(F \cup G) = (F \cup E)(F \cup G) = F \cup EG$. (b). Using part (a), we have $(E \cup F)(E^c \cup F)(E \cup F^c) = (F \cup EE^c)(E \cup F^c) = F(E \cup F^c) = FE \cup FF^c = FE$.

S1.4-6: The probability that the first horse wins is $2/7$. The probability that the second horse wins is $3/10$. Since the events that the first horse wins and the second horse wins are mutually exclusive, the probability that either the first horse or the second horse will win is $2/7 + 3/10 = 41/70$.

S1.4-18: Let M and F denote the events that the randomly selected student earned an A on the midterm exam and an A on the final exam, respectively. Then

$$\mathbb{P}(MF) = \mathbb{P}(M) + \mathbb{P}(F) - \mathbb{P}(M \cup F),$$

where $\mathbb{P}(M) = 17/33$, $\mathbb{P}(F) = 14/33$, and by DeMorgan's law,

$$\mathbb{P}(M \cup F) = 1 - \mathbb{P}(M^c F^c) = 1 - 11/33 = 22/33.$$

Therefore, $\mathbb{P}(MF) = 17/33 + 14/33 - 22/33 = 3/11$.

S1.4-22: Let T and F be the events that the number selected is divisible by 3 and 5, respectively.
(a). The desired quantity is the probability of the event TF^c:

$$\mathbb{P}(TF^c) = \mathbb{P}(T) - \mathbb{P}(TF) = 333/1000 - 66/1000 = 267/1000.$$

(b). The desired quantity is the probability of the event T^cF^c:

$$\begin{aligned} \mathbb{P}(T^cF^c) &= 1 - \mathbb{P}(T \cup F) = 1 - \mathbb{P}(T) - \mathbb{P}(F) + \mathbb{P}(TF) \\ &= 1 - \frac{333}{1000} - \frac{200}{1000} + \frac{66}{1000} = \frac{533}{1000}. \end{aligned}$$

R1-10. NO. It is 1 in 4. Three ways to see it:
(i). list sample spaces;
(ii). look at a fixed student: (s)he has to pick the tire that the other picked;
(iii). $\mathbb{P}(\text{same tire}) = \sum_{k=1}^{4} \mathbb{P}(\text{ same k-th tire}) = \sum_{k=1}^{4} \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4}$
R1-14: Let E, F, G, and H be the events that the next baby born in this town has blood type O, A, B, and AB, respectively. Then

$$\mathbb{P}(E) = \mathbb{P}(F),\ \mathbb{P}(G) = (1/10)\mathbb{P}(F),\ \mathbb{P}(G) = 2\mathbb{P}(H).$$

These imply $\mathbb{P}(E) = \mathbb{P}(F) = 20\mathbb{P}(H)$. Therefore, from $\mathbb{P}(E)+\mathbb{P}(F)+\mathbb{P}(G)+\mathbb{P}(H) = 1$, we get $20\mathbb{P}(H) + 20\mathbb{P}(H) + 2\mathbb{P}(H) + \mathbb{P}(H) = 1$, which gives $\mathbb{P}(H) = 1/43$.