

The Gaussian Correlation Conjecture

The standard Gaussian measure on \mathbb{R}^n is given by

$$\gamma(A) = \gamma_n(A) = \frac{1}{(2\pi)^{n/2}} \int_A e^{-|x|^2/2} dx.$$

A general mean zero Gaussian measure on \mathbb{R}^n is just a linear image of the standard Gaussian measure.

Conjecture (GCC). . For any $n \geq 1$, if μ is a mean zero, Gaussian measure on \mathbb{R}^n , then for any $A, B \subseteq \mathbb{R}^n$, which are convex and symmetric about the origin,

$$\mu(A \cap B) \geq \mu(A)\mu(B).$$

A much weaker version of the problem was considered in 1955 and appeared in its present form about 1970. The original application was to (multidimensional) confidence regions in Statistical problems.

In addition to some background we will discuss some new approaches to this conjecture.