

ON INTEGRATED RANDOM WALKS THAT STAY POSITIVE

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Let S_n be a centered random walk with a finite variance, and define the new sequence $\sum_{i=1}^n S_i$, which we call the *integrated random walk*. We are interested in the asymptotics of

$$p_N := \mathbb{P} \left\{ \min_{1 \leq k \leq N} \sum_{i=1}^k S_i \geq 0 \right\}$$

as $N \rightarrow \infty$. Sinai (1992) proved that $p_N \asymp N^{-1/4}$ if S_n is a simple random walk. We prove that p_N has the same asymptotics for some other types of random walks, including Laplace walks. We also show that $p_N \leq cN^{-1/4}$ for every lattice walk and certain continuous walks.