

Strong Approximations to the smoothing Spline With Applications to Confidence Bands

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Abstract

Strong approximations by simple Gaussian Processes are established for the smoothing spline estimator (minus its expectation). The result is shown to hold for splines of arbitrary order, and for both deterministic and data driven selections of the smoothing parameter. For the case of a uniform design, the result provides a simple method for determining the asymptotic distribution of test statistics and pivot variables, based on the spline estimator. This is illustrated by determining asymptotic confidence bands for the mean function.

For non uniform designs the strong approximation still holds, but the resulting Gaussian process is not stationary! Further, while not known exactly the associated covariance kernel is related to the Green's function for a given boundary value problem. The unanswered question raised in this talk is can the distribution of the sup norm of the associated Gaussian Process be determined?

As is typical for such problems, even when known, the rate of convergence of the distribution of the pivot variable, upon which the confidence band is based, to its asymptotic form is quite slow. A simple approximation to the distribution of the pivot random variable is developed, which improves the accuracy of the confidence bands for moderate size samples.

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