A Constrained ℓ_1 Minimization Approach to Sparse Precision Matrix Estimation

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Abstract

A constrained ℓ_1 minimization method is proposed for estimating a sparse inverse covariance matrix based on a sample of n iid p-variate random variables. The resulting estimator is shown to enjoy a number of desirable properties. In particular, it is shown that the rate of convergence between the estimator and the true s-sparse precision matrix under the spectral norm is $s\sqrt{\log p/n}$ when the population distribution has either exponential-type tails or polynomial-type tails. Convergence rates under the elementwise ℓ_∞ norm and Frobenius norm are also presented. In addition, graphical model selection is considered. The procedure is easily implementable by linear programming. Numerical performance of the estimator is investigated using both simulated and real data. In particular, the procedure is applied to analyze a breast cancer dataset. The procedure performs favorably in comparison to existing methods.