

M241 -- PRACTICE TEST 3 -- Fall 1999 -- WENGER

[Note: This test during this semester covered section 3.4, 3.5, 3.6, 3.7, 4.1, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 5.1, 5.2, 5.3 of Thomas/Finney, 9th Edition] \prac_T3.pdf

Note: This is only a sample test. Other topics were covered in the sections covered which may appear on the actual test. The actual test would not be this long, of course.

You must clearly justify all your conclusions and show all work used to arrive at them.

1. Determine the vertical and horizontal asymptotes for $g(x) = \frac{3x + 1 - \frac{1}{x^2}}{x^2 - 1}$.
2. Determine the absolute minimum and absolute maximum for $h(x) = x^3 - 12x$ on the interval $[0,3]$.
3. Determine dy/dx if $y = \int_0^{\sin(x)} t^3 dt$.
4. Integrate each of the following:
- a) $\int \sec(3x + 2)^2 dx$
- b) $\int x^{\left(\frac{1}{2}\right)} \sin\left(x^{\left(\frac{3}{2}\right)} + 1\right) dx$
- c) $\int_0^{\frac{\pi}{4}} (1 - \sin(2t))^{\left(\frac{3}{2}\right)} \cos(2t) dt$

d) $\int_{\pi}^{\frac{3\pi}{2}} \tan\left(\frac{x}{6}\right)^5 \csc\left(\frac{x}{6}\right)^2 dx$

5. Create definite integrals whose values would equal the area of the region in the first quadrant enclosed by the curve $y = -x^2 + 6$ and $y = 5x$ using:

a) integration with respect to x .

b) integration with respect to y .

6. The function $f(x) = \frac{8}{x^2 + 4}$ has $\frac{df}{dx} = -\frac{16x}{(x^2 + 4)^2}$ and $d^2f/dx^2 = 16\frac{3x^2 - 4}{(x^2 + 4)^3}$.

With this information, answer the following question.

a) What is the domain of the function f ? What are its vertical and horizontal asymptotes, if any?

b) Identify the intervals where f is increasing and decreasing.

c) What are the points of local maxima and minima?

d) Identify the intervals where f is concave up and concave down.

e) What are the inflection points of f ?

7. Determine the linearization for $f(x) = (x + 1)^2 + \tan(x)$ at $x = 0$.

8. Determine the average value of $f(x) = 3x^2 - 3$ on $[0,1]$.

9. Determine the volume of the solid generated by revolving the region bounded by $y = \sec(x)$, $y = 0$, $x = 0$, and $x = \frac{\pi}{4}$ about the x -axis.

10. A solid with its base on the xy -plane lies between the planes perpendicular to the x -axis at $x = 0$ and $x = 1$. The cross section perpendicular to the x -axis are squares whose sides run from $y = 4x^2$ to $y = 9 - 5x^2$. Express the volume as a definite integral.

11. Determine the point on the curve $y = 2x^{\left(\frac{3}{2}\right)}$, $7/9 \leq x \leq 8/3$, which is nearest to the point (5,0).
12. a) Determine the specific values for c_k and each of the terms $f(c_k) \Delta x$ in the Riemann sum $\sum_{k=0}^3 f(c_k) \Delta x$ if the *left* endpoints are used for $f(x) = x^2$ on $[-1, 3]$ using subintervals of equal length.
- b) Sketch the graph of f and the rectangles used in part a). Clearly indicate the values used on the x- and y-axes.
13. State the Fundamental Theorem of Calculus.