0. a17.1-9

1. (a17.1) Find the Lagrange interpolation polynomial which fits the data by
   (1) solving equations for unknown coefficients,
   (2) Lagrange nodal basis,
   (3) Newton’s divided differences.

   \[
   \begin{array}{c|cccc}
   x_i & 0 & 2 & 3 \\
   y_i & 1 & 3 & 0 \\
   \end{array}
   \]

2. (a17.2) Find the Lagrange interpolation polynomial which fits the data by
   (1) solving equations for unknown coefficients,
   (2) Lagrange nodal basis,
   (3) Newton’s divided differences.

   \[
   \begin{array}{c|cccc}
   x_i & -1 & 0 & 1 & 2 \\
   y_i & 0 & -2 & 0 & 0 \\
   \end{array}
   \]

3. (a17.3) Find the Lagrange interpolation polynomial which fits the data by
   (1) solving equations for unknown coefficients,
   (2) Lagrange nodal basis,
   (3) Newton’s divided differences.

   \[
   \begin{array}{c|cccc}
   x_i & -1 & 0 & 1 & 2 \\
   y_i & 0 & 0 & 0 & 6 \\
   \end{array}
   \]

4. (a17.4) Find the Lagrange interpolation polynomial which fits the data by
   (1) solving equations for unknown coefficients,
   (2) Lagrange nodal basis,
   (3) Newton’s divided differences.

   \[
   \begin{array}{c|cccc}
   x_i & -1 & 2 & 3 & 5 \\
   y_i & 0 & 1 & 1 & 2 \\
   \end{array}
   \]

5. (a17.5) By the method of Newton’s divided differences, find the interpolation the function by a degree 3 polynomial
   \( P(x) \): at points

   \[
   \begin{array}{c|cccc}
   x & -1 & 1 & 2 & 3 \\
   y & 3 & 3 & 3 & 7 \\
   \end{array}
   \]

   For \( P_d, \ d = 2, 3, 6 \), how many polynomials interpolate the function at the 4 points?

6. (a17.6) Find \( P_{10}(0) \) interpolating \( f(x) \) at the given points,

   \[
   f(x_i) = 0, \quad x_i = 1, 2, ..., 10, \\
   f(12) = 44
   \]

7. (a17.7) Let \( f(x) = 6x^3 - 4x \). Let \( x_i = 0, 1, 2 \)

   (a) Find the Lagrange interpolation polynomial which fits the data at the three points by solving the linear system of unknown coefficients.
   (b) Find the Lagrange interpolation polynomial by the Lagrange nodal basis.
   (c) Find \( f(1.5), P_2(1.5) \), the error \( f(1.5) - P_2(1.5) \), and the relative error.

8. (a17.8) Let \( f(x) = \sqrt{2x + 1} \).
   (a) Find \( f(4), f'(4), \) and \( \int_0^4 f(x)dx \).
   (b) Find the degree 2 Taylor polynomial, \( T_2(x) \), of \( f(x) \) at \( x_0 = 0 \).
   (c) Find \( T_2(4), T_2'(4), \) and \( \int_0^4 T_2(x)dx \).
   (d) Find the degree 2 interpolation polynomial, \( P_2(x) \), of \( f(x) \) with interpolation points \( x_0 = 0, x_1 = 1, \) and \( x_2 = 2 \).
   (e) Find \( P_2(4), P_2'(4), \) and \( \int_0^4 P_2(x)dx \).
   (f) Find all relative errors. Which polynomial is better in approximating the three wanted values?

9. (a17.9) Let \( f(x) = (x + 1)^{1/3} \).
   (a) Find the degree 3 Taylor polynomial, \( T_3(x) \), of \( f(x) \) at \( x_0 = 0 \).
   (b) Find the relative errors, when approximating \( f(3), f'(3), \) \( \int_0^3 f(x)dx \) by \( T_3(3), T_3'(3), \) \( \int_0^3 T_3(x)dx \)
   (c) Find the degree 3 interpolation polynomial, \( P_3(x) \), of \( f(x) \) with interpolation points \( x_0 = 0, x_1 = 1, x_2 = 2 \) and \( x_3 = 3 \).
   (d) Find the relative errors, when approximating \( f(3), f'(3), \) \( \int_0^3 f(x)dx \) by \( P_3(3), P_3'(3), \) \( \int_0^3 P_3(x)dx \)
   (e) Which polynomial seems to be better in approximating the three values wanted?