0. a10.1-6, 10.3-5, 8, 10,11, 14 (no MATLAB)

1. (2.2:a1) (1) Find $A = LU$ decomposition. (2) Use $LU$ decomposition of $A$ to solve $Ax = b$ in 2 steps. (3) Do GE without pivoting, to solve $Ax = b$ again.

   \[
   (A|b) = \begin{pmatrix}
   1 & 0 & 1 & 2 & | & 1 \\
   2 & -3 & 5 & -2 & | & -2 \\
   1 & -3 & 3 & -2 & | & 0 \\
   0 & 4 & -11/2 & 12 & | & 1
   \end{pmatrix}
   \]

   Hint: You may follow the method in the lecture – doing (3) first, then answering (1) & (2).

2. (a10.2) Do both methods: (1) Direct GE with PP. (2) Find $PA = LU$ and use it and 2-step substitution to solve $Ax = b$.

   \[
   A = \begin{pmatrix}
   4 & 2 & 0 \\
   4 & 4 & 2 \\
   2 & 2 & 3
   \end{pmatrix}, \quad b = \begin{pmatrix}
   2 \\
   4 \\
   6
   \end{pmatrix}
   \]

3. (a10.4) Solve the following system $(A|b)$

   (a) Find $PA = LU$ decomposition.
   (b) Find $A = LU$ decomposition.
   (c) GE without pivoting, (no row switching, no row multiplication)
   (d) Use $A = LU$ and 2-step substitution method.
   (e) GE with partial pivoting, (row pivoting every step)
   (f) Use $PA = LU$ and 3-step substitution method.

   \[
   \begin{pmatrix}
   1 & 2 & 3 & | & 2 \\
   2 & 1 & -1 & | & 0 \\
   3 & 0 & -3 & | & 0
   \end{pmatrix}
   \]

4. (a10.3) Solve the following system $(A|b)$ by GE with

   (a) without pivoting, (no row switching, no row multiplication). Then show the $A = LU$ decomposition and 2-step substitution method for solving the problem.
   (b) with partial pivoting, (row pivoting every step) Then show the $PA = LU$ decomposition and 3-step substitution method for solving the problem.

   \[
   \begin{pmatrix}
   1 & -3 & 2 & | & 5 \\
   -2 & 0 & 2 & | & 2 \\
   3 & -1 & -1 & | & 0
   \end{pmatrix}
   \]

5. (a10.5) Find the Cholesky factorization and use it to solve $Ax = b$ by two-step substitution.

   \[
   A = \begin{pmatrix}
   4 & 0 & -2 \\
   0 & 1 & 1 \\
   -2 & 1 & 3
   \end{pmatrix}, \quad b = \begin{pmatrix}
   4 \\
   2 \\
   0
   \end{pmatrix}
   \]

6. (a10.6) Find the Cholesky factorization $A = R^T R$ and use it to solve $Ax = b$ by two-step substitution.

   \[
   A = \begin{pmatrix}
   16 & 4 & 0 & -4 \\
   4 & 5 & 2 & -1 \\
   0 & 2 & 2 & -2 \\
   -4 & -1 & -2 & 6
   \end{pmatrix}, \quad b = \begin{pmatrix}
   8 \\
   2 \\
   -4 \\
   3
   \end{pmatrix}
   \]