M353 8.3 Elliptic (S. Zhang) .

1. (8.3:a1) Solve the finite difference equations for the Laplace equation

$$u_{xx} + u_{yy} = 0$$

in the rectangular region $0 \le x \le 2,\ 0 \le y \le 3$ with the boundary condition:

$$\begin{aligned} u(0,y) &= 0, & u(2,y) &= 8y \\ u(x,0) &= 0, & u(x,3) &= 12x \end{aligned}$$

Assume $h = 1$. (Exact solution $u = 4xy$.)

2. (8.3:a2) Solve the finite difference equations for the Laplace equation with $h = 1$.

$$u_{xx} + u_{yy} = 2 + 2x,\ 0 \le x, y \le 2$$

with the boundary condition

$$\begin{aligned} u_x(0,y) &= y^2, & u(2,y) &= 2y^2 + 4 \\ u(x,0) &= x^2, & u(x,2) &= 4x + x^2. \end{aligned}$$

3. (8.3:a3) Solve the finite difference equations for the Laplace equation with $h = 1$.

$$u_{xx} + u_{yy} = 2x + 4y,\ 0 \le x, y \le 2$$

with the boundary condition

$$\begin{aligned} u_x(0,y) &= y^2, & u(2,y) &= 2y^2 + 8y \\ u_y(x,0) &= 2x^2, & u(x,2) &= 4x^2 + 4x. \end{aligned}$$

4. (8.3:a4) Solve the finite difference equations for the Laplace equation with $h = 1$.

$$u_{xx} + u_{yy} = 4 + 2x,\ 1 \le x \le 2,\ 0 \le y \le 2$$

with the boundary condition

$$\begin{aligned} u_x(1,y) &= 4 + y^2, & u_x(2,y) &= 8 + y^2 \\ u(x,0) &= 2x^2, & u(x,2) &= 4x + 2x^2. \end{aligned}$$

Then, again, with $h = 1/2$.