M353 8.1 Heat (S. Zhang) .

1. (8.1:a1) Solving the heat equation by the finite difference method. $u_t = 4u_{xx}$ for $t \in (0, 0.1)$ and $x \in (0, 3)$ with initial and boundary values

$$u(x,0) = 2(3-x)x, \quad u(0,t) = 0, \quad u(3,t) = 0.$$

Let $\Delta t = 0.1$. Find $u(x_i, 0.1)$ by the following methods with $h = 1$.

(a) the Euler (explicit) method.

(b) the backward Euler (implicit) method.

(c) the Crank-Nicholson method.

2. (8.1:a2) Solving the heat equation by the finite difference method. $u_t = 4u_{xx}$ for $t \in (0, 0.1)$ and $x \in (0, 1)$ with initial and boundary values

$$u(0,t) = 0, \quad u(1,t) = 0.\text{(initial conditions)}$$
$$u(x,0) = 2x, \quad \text{(BC, use } x(1,0) = 0 \text{ from the IC above)}$$

Let $k = \Delta t = 0.1$. Find $u(x_i, 0.1)$ by the following methods with $h = 1/2$ and with $h = 1/3$ again.

(a) the Euler (explicit) method .

(b) the backward Euler (implicit) method.

(c) the Crank-Nicholson method.

3. (8.1:a3) Solving the heat equation by the finite difference method. $u_t = u_{xx}$ for $t \in (0, 0.2)$ and $x \in (1, 5)$ with initial and boundary values

$$u(x,0) = (5-x)(1-x), \quad u(1,t) = 0, \quad u(5,t) = 0.$$

(a) the Euler (explicit) method. $h = 1$ and $k = 0.1$.

(b) the backward Euler (implicit) method. $h = 1$ and $k = 0.1$.