M353 7.1-2 Shooting/FD (S. Zhang) .

1. (7.1-2:a1) Solve the boundary value problem by the Euler shooting method with $h = 1/2$ (find approximate values $x(1/2)$) :

$$x'' - x' = 2 - 2t, \qquad x(0) = 2, \quad x(1) = 3$$

Then solve it by the finite difference method with $h = 1/2$, and $h = 1/3$. The exact solution is given:

$$x = t^2 + 2$$

2. (7.1-2:a2) Solve the boundary value problem by the Euler shooting method with $h = 1$ (find approximate values $x(1)$) :

$$x'' - x' - 2x = -2t - 5, \qquad x(0) = 2, \quad x(2) = 4$$

Then solve it by the finite difference method with $h = 1$, and $h = 1/2$. Extrapolate the solutions here.

3. (7.1-2:a3) Solve the boundary value problem by shooting method:

$$x'' - 3x' + 2x = t + 1,$$
$$x(0) = 1, \quad x(1) = 3$$

   (a) Find the exact solution. $(x_H + x_P)$

   (b) Find the exact solution $u = x(t)$ for (shooting 1)

$$x'' - 3x' + 2x = t + 1, \qquad x(0) = 1, \qquad x'(0) = 0$$

   (c) Find the exact solution $v = x(t)$ for (shooting 2)

$$x'' - 3x' + 2x = 0 \qquad x(0) = 0, \qquad x'(0) = 1$$

   (d) Then combine u and v by the shooting method to get the exact solution.

   (e) Convert the above two shooting problems to systems of two equations, and apply the Euler method with $h = 1/2$ to them. Combine the two discrete solutions and find the approximate value of $x(1/2)$. Find the error.

   (f) Solve the BVP by the Euler shooting method with $h = 1/4$. Find the error at $t = 1/2$.

   (g) Solve the BVP by the Runge-Kutta shooting method with $h = 1/2$. Find the error at $t = 1/2$.

   (h) Solve the finite difference equations for the boundary value problem with grid size $h = 1/2$ to get an approximation of $x(1/2)$. Find the error.

   (i) Solve the BVP by the finite difference method with $h = 1/4$. Find the error at $t = 1/2$. Use extrapolation on the two $x(1/2)$ values to find a new $x(1/2)$ and the error.

4. (7.2:a5) Solve the boundary value problem (finding $x(1)$):

$$x'' + x = e^x, \qquad x(0) = 1, \quad x(2) = 4$$

   (a) by the Euler **nonlinear** shooting method with $h = 1$ and 2 steps (5 shootings) of the bisection method on $x'(0) \in [0, 2]$.

   (b) by the finite difference method with $h = 1$; with 4 steps of the Newton's method starting with $x_1 = 2$.