

**M353 2.5 Jacobi** (S. Zhang) .

1. (2.5:a1) Find  $x_2$  by (a) Jacobi and by (b) Gauss–Seidel iterations, find (c)  $\|R_j\|_\infty$ , (d)  $\|R_{gs}\|_\infty$ , and (e) verify the error bounds  $\|x - x_i\|_\infty \leq \|R_j\|_\infty^i \|x - x_0\|_\infty$ .

$$A = \begin{pmatrix} 2 & 0 \\ -1 & 2 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

2. (2.5:a2) Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}, b = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}, x_0 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

- (a) Find  $x_2$  if Jacobi iteration is used.
- (b) Find  $x_2$  if Gauss–Seidel iteration is used.
- (c) Show  $A$  is strictly diagonally dominated.
- (d) Find the error reduction factor for the Jacobi iteration,  $\|R_j\|_\infty$ .
- (e) Find the error reduction factor for the Gauss–Seidel iteration,  $\|R_{gs}\|_\infty$ .