**M353 Study Guide 3** (S. Zhang) .

1. Compute $\int_0^2 5x^4\,dx$.

   (a) Use the trapezoidal rule with $m = 1 (h = 2)$, and $m = 2$. Find the extrapolation. Find all three errors.

   (b) Use the mid-point rule with $m = 1 (h = 2)$, and $m = 2$. Find the extrapolation. Find all three errors.

   (c) Use the Simpson's rule with $m = 1 (h = 1)$, and $m = 2$. Find the extrapolation. Find all three errors.

2. Find Romberg $R_{33}$ and the error for

$$\int_1^3 6x^5\,dx.$$

3. Determine the smallest integer $j$ and $N$, for which the Romberg integral $R_{jj}$ (starting from $R_{11}$) and the Gauss-Legendre integral $G_N$ is the exact, respectively.

$$\int_2^{12} 77x^{14} - x^3\,dx$$

4. Compute the integral by adaptive quadrature with tolerance 0.2

$$\int_1^2 4x^3\,dx.$$

5. Compute Gauss integration $G_2$, $G_3$ ( $x_i = \pm\sqrt{.6}, 0$, $c_i = \frac{5}{9}, \frac{8}{9}$ ) for

$$\int_1^2 x^4 + 1\,dx.$$

6. Solve the IVP. Find $y(0.5)$,

$$y' + 2y = 2e^{-t}; \quad y(0) = 2.$$

   Apply Euler method with $h = 1/2$ and $h = 1/4$. Extrapolate the solutions for $y(1/2)$. Find three errors.

7. Given an initial value problem

$$x'' - 4x = 8t, \quad 0 < t < 0.2$$
$$x(0) = 0, \quad x'(0) = -2.$$

   (a) Find the exact solution.

   (b) Convert the equation to a system. Use Euler's method with step size $h = 0.1$ and find the error.

   (c) Use R-K method with step size $h = 0.2$ and find the error.

8. Find the exact solution. Apply backward Euler with $h = 1/4$ for $y(1)$, find the error.

$$y' = t - y, \; y(0) = 0$$

9. Solve the boundary value problem (BVP) by shooting method:

$$x'' - 3x' + 2x = 4t^2 - 12t + 6,$$
$$x(0) = 1, \quad x(1) = 3$$

   (a) Find the exact solution. $(x_H + x_P)$

   (b) Find the exact solution $u = x(t)$ for IVP (shooting 1)

$$x'' - 3x' + 2x = 4t^2 - 12t + 6, \quad x(0) = 1, \quad x'(0) = 0$$

   (c) Find the exact solution $v = x(t)$ for IVP (shooting 2)

$$x'' - 3x' + 2x = 0 \qquad x(0) = 0, \qquad x'(0) = 1$$

   (d) Then combine $u$ and $v$ by the shooting method to get the exact solution of the original BVP.

   (e) (Linear Euler Shooting) Convert the above two shooting problems to systems of two equations, and apply the Euler method with $h = 1/2$ to them. Combine the two discrete solutions and find the approximate value of $x(1/2)$. Find the error.

   (f) (Linear Euler Shooting) Solve the BVP by the Euler shooting method with $h = 1/4$. Find the error at $t = 1/2$. Extrapolate two linear shooting results to get a better approximation of $x(1/2)$.

   (g) (Linear shooting) Solve the BVP by the Runge-Kutta shooting method with $h = 1/2$. Find the error at $t = 1/2$.

   (h) ( **Nonlinear** Euler shooting ) Apply the 2 steps (5 shootings) of the bisection method with Euler discretization of grid size $h = 1$, using starting interval $x'(0) \in [-1, 1]$. Find approximate $x(1/2)$ and its error.

   (i) Solve the finite difference equations for the boundary value problem with grid size $h = 1/2$ to get an approximation of $x(1/2)$. Find the error.

   (j) Solve the BVP by the finite difference method with $h = 1/4$. Find the error at $t = 1/2$.

   (k) Use extrapolation on the last two $x(1/2)$ values to find a new $x(1/2)$ and the error.

10. Solve the boundary value problem by the Euler shooting method with $h = 1/2$ (find approximate values $x(1/2)$ ) :

$$x'' - x' = 2 - 2t, \qquad x(0) = 2, \quad x(1) = 3$$

   Then solve it by the finite difference method with $h = 1/2$, and $h = 1/3$. The exact solution is given:

$$x = t^2 + 2$$