M353 Study Guide 1 (S. Zhang) .

1. Find the following sums by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule:

$$(a)(1 + (2^{-51} + 2^{-52} + 2^{-54})) - 1$$
$$(b)(1 + (2^{-51} + 2^{-52} + 2^{-60})) - 1$$

2. Convert $a = 4.125$ and $b = 19/7$ to IEEE doubles. Then find the IEEE double form for $a + b$ (using chopping after bit 52.)

3. Find an interval of length one that contains a root.

$$x^4 = x^2 + 2$$

Then do two steps of bisection method. How many steps of bisection iterations are need to reach 10^{-10} accuracy?

4. Express the equation as a fixed point iteration in 3 ways.

$$x^4 + 9x^3 = x^2$$

Find the fixed point iteration for the Newton's method.

5.

$$g(x) = -4 + 4x - \frac{1}{2}x^2$$

   (a) Find fixed points.
   (b) Find the local convergence at each fixed point.
   (c) Do 2 steps of fixed point iteration with $p_0 = 1.9$.
   (d) Do 2 steps of fixed point iteration with $p_0 = 3.8$.
   (e) Find errors above.
   (f) What about the convergence?

6. Find convergence order and the rate of the Newton's method

$$32x^3 - 32x^2 - 6x + 9 = 0, \; r = -1/2, 3/4$$

7. For finding the root of the function:

$$f(x) = x^2 - 5x - 11$$

   (a) Do 3 steps of the Newton's method, $p_0 = 6$. Find the errors and use the data to show the method is a second-order one.
   (b) Do 3 steps of the secant method, $p_0 = 0$, $p_1 = 8$.
   (c) Do 3 steps of the false position method, given the initial interval $[0, 8]$.

8. Find $\det(A)$ by expansions directly. Then again, by reducing A to an upper triangular matrix.

$$A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 2 & 2 & -1 \\ 2 & 2 & 5 & 0 \end{pmatrix}$$

9. Find A^{-1} by elementary row operations,

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & -1 & 3 & 3 \\ -1 & 0 & -2 & 0 \\ 0 & 3 & -4 & 6 \end{pmatrix}$$

10. Let

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 4 & -3 & 4 \\ -6 & 1 & 1.5 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 8 \\ -4.5 \end{pmatrix}.$$

   (a) Use Gaussian elimination without pivoting to solve $Ax = b$.
   (b) Find $A = LU$ where L is the unit lower triangular matrix.
   (c) Using LU decomposition of A to solve $Ax = b$.
   (d) Use Gaussian elimination with partial pivoting to solve $Ax = b$.

11. Find a nonzero vector x so that $\|Ax\|_\infty = \|A\|_\infty \|x\|_\infty$

$$A = \begin{pmatrix} 1 & -3 & 2 \\ 3 & 1 & -3 \\ 2 & 1 & 0 \end{pmatrix}$$

Can we find a nonzero x so that $\|Ax\|_\infty = 2\|A\|_\infty \|x\|_\infty$?

12. Find the relative forward and backward errors, and error magnification factors. Verify that the factors are no bigger than the condition number.

$$(A|b) = \left(\begin{array}{cc|c} 1 & 2 & -1 \\ 2 & -3 & 5 \end{array}\right)$$

$$(a)\; x_c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad (b)\; x_c = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

13. Solve the following system $(A|b)$

$$\left(\begin{array}{ccc|c} 1 & -3 & 3 & 4 \\ -2 & 0 & 1 & -1 \\ 3 & -1 & -1 & 2 \end{array}\right)$$

   (a) by GE without pivoting,
   (b) by finding $A = LU$ and using it
   (c) by GE with partial pivoting,
   (d) by finding $PA = LU$ and using it

14. Solve the following system $(A|b)$

$$\left(\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ -1 & 3 & -2 & 6 \end{array}\right)$$

   (a) by GE without pivoting,
   (b) by finding $A = LU$ and using it
   (c) by GE with partial pivoting, (row pivoting every step)
   (d) by finding $PA = LU$ and using it