M353 Hw 8 (S. Zhang) 5.5 .

1. 5.5: 1cd, 2cd, 4cd, 5cd, 7, 9,a1

64.99

1. (5.5:4c5.5:5c) Compute Gauss integrations G_2 and G_3
64.202 , for

$$\int_0^1 xe^x\,dx.$$

Find the error.

- **ans:**

$$\int_a^b f(z)dz = \int_{-1}^1 f(\frac{a+b}{2} + t\frac{b-a}{2})\frac{b-a}{2}dt$$

$$\begin{aligned}\int_0^1 xe^x\,dx = \int_{-1}^1 \frac{1+t}{2}e^{\frac{1+t}{2}}\frac{1}{2}dt\\ =&F(-\frac{1}{\sqrt{3}}) + F(\frac{1}{\sqrt{3}})\\ =&\frac{1-\frac{1}{\sqrt{3}}}{2}e^{\frac{1-\frac{1}{\sqrt{3}}}{2}} + \frac{1+\frac{1}{\sqrt{3}}}{2}e^{\frac{1+\frac{1}{\sqrt{3}}}{2}}\\ =&\frac{0.2611}{2} + \frac{1.7355}{2}\\ =&0.9983\end{aligned}$$

Exact integral:

$$\int_0^1 xe^x\,dx = (x+1)e^x|_0^1 = 0 - (-1) = 1$$

Error

$$\int_0^1 xe^x\,dx - G_2 = 0.0017$$

$$G_3:\ x_i = \pm\sqrt{.6}, 0;\ c_i = \frac{5}{9}, \frac{8}{9}$$

$$F(t) = \frac{1+t}{2}e^{\frac{1+t}{2}}\frac{1}{2}$$

$$\begin{aligned}G_3 =&\frac{5}{9}F(-\sqrt{.6}) + \frac{8}{9}F(0) + \frac{5}{9}F(-\sqrt{.6})\\ =&\frac{5}{9}0.063073371949896 + \frac{8}{9}0.412180317675032\\ &+\frac{5}{9}1.077428455314857\\ =&0.999994630858225\end{aligned}$$

$$err = 1 - .99999463085 = 0.0000053$$

2. (5.5:a1) (1) Determine the smallest N for which the
64.38 Gauss-Legendre integral G_N is exact.
(2) Determine the smallest J for which the Romberg integral R_{JJ} is exact.

$$(a)\ \int_0^{2\pi}(8x^7 - 1)\,dx$$

$$(b)\ \int_1^{\sqrt{2}}(11x^{10} + \sqrt{5}x)\,dx$$

- **ans:** (1) For Gauss-Legendre integrals, G_N are exact for polynomial of degree $2N-1$.

(a)

$$2N - 1 \geq 7,\ N = 4$$

(b)

$$2N - 1 \geq 10,\ N = 6$$

(2) For Romberg integrals, R_{JJ} is exact for is exact for polynomial of degree $2J-1$. R_{11} is exact for P_1 and each time it increases the degree by 2.

(a)

$$2J - 1 \geq 7,\ J = 4$$

(b)

$$2J - 1 \geq 10,\ J = 6$$
