

M351 Study Guide 3 (S. Zhang).

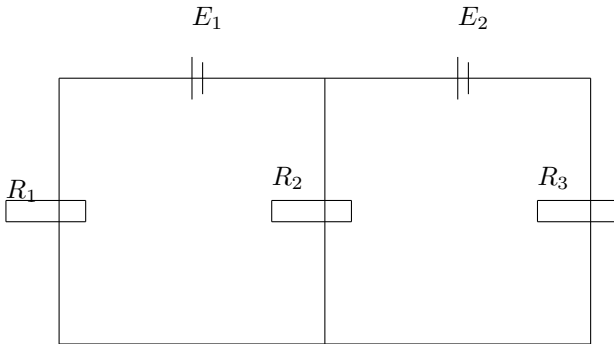
1. Find counter-examples:

$$AB \neq BA, (A+B)(A-B) \neq A^2 - B^2$$

2. Find $B^T(2A)$ and verify $A^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$, if

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

3. Currents in a network. $R_1 = 3\text{ohms}$, $R_2 = 5$, $R_3 = 6$, $E_1 = 10\text{volts}$, $E_2 = 27\text{volts}$, . Find currents.



4. Solve the linear system by

- (1) Gaussian elimination (must reach Row-echelon Form)
- (2) Gauss-Jordan elimination (reach Reduced Row-echelon Form)

$$\begin{array}{rclcl} x_1 & +2x_2 & -4x_3 & = & 9 \\ 5x_1 & -x_2 & +2x_3 & = & 1 \end{array}$$

5. (1) Determine if $(A|b)$ is in row-echelon form, reduce it to a row-echelon form if not, and use its row-echelon form to solve $Ax = b$.

(2) Determine if $(A|b)$ is in reduced row-echelon form, reduce it to a THE reduced row-echelon form if not, and use THE row-echelon form to solve $Ax = b$.

$$(a) \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$(b) \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$(c) \left(\begin{array}{cccc|c} 0 & 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$(d) \left(\begin{array}{cccc|c} 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right)$$

6. Find the linearly dependence by rank and the number of free variables for the homogeneous system $Ax = 0$.

$$(1) \mathbf{u}_i = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix},$$

$$(2) \mathbf{u}_i = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

$$(3) \mathbf{u}_i = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$(4) \mathbf{u}_i = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$$

7. Find the determinant

- (1) by cofactor expansions (no row/column operations)
- (2) by row operations to an upper triangular matrix.
- (3) by smart combinations of row/column op and expansion.

$$A = \begin{pmatrix} 2 & 0 & 4 & 4 \\ 5 & -10 & 5 & 5 \\ 0 & 0 & 4 & 4 \\ 1 & 2 & 0 & 2 \end{pmatrix}$$

8. $|A_{4 \times 4}| = -2$, $|B_{4 \times 4}| = -1$, find

$$|2A^{-1}|, |A^T B|, |-2B^{-1}|, |B^2|$$

9. Find A^{-1} by (1) cofactors, (2) row operations. And use A^{-1} to solve $Ax = b$.

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

10. Solve $Ax = b$ by Cramer's rule

$$(1) A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(2) A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix}$$

11. Find eigenvalues and eigenvectors.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{pmatrix}$$

12. Find eigenvalues and eigenvectors.

$$\begin{pmatrix} -1 & 2 \\ -5 & 1 \end{pmatrix}$$

13. Find Orthogonal eig-matrices:

$$A = \begin{pmatrix} 1 & 9 \\ 9 & 1 \end{pmatrix}, A = \begin{pmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

14. Factor the matrix A into a product DXD^{-1} , where D is diagonal. Then find A^{132} by the factorization.

$$A = \begin{pmatrix} -2 & -2 \\ 1 & 1 \end{pmatrix}; A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

15. Find possible values for a that make the matrix defective, i.e., A is not diagonalizable, or show that no such values exist.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & a \end{pmatrix}$$