M351 Study Guide 2 (S. Zhang) .

1. Solve
$$y'' - 16y = 2e^{4x}$$

2. Solve
$$y'' - 2y' - 3y = 3e^{2t}.$$

3. Solve IVP
$$y'' + y = 2\sin t, \quad y(0) = 0,\ y'(0) = -1.$$

4. Find the general solution of the homogeneous equation y_c and write down the form for undetermined coefficients for y_p. (Do not solve for y_p.)
$$(D^2 - 9)(D+3)(D^2+4)y = e^x \sin x - e^{-3x} + \cos 2x.$$

5. Find the general solution by both the undetermined coefficients method and the variation of parameters method.
$$y'' - y = \cos^2 x.$$

6. Solve
$$y'' + y = \tan x$$

7. Solve
$$y''' + y' = \tan x$$

8. Solve an Euler equation:
$$(x-1)^2 y'' + 8(x-1)y' + 12y = 0$$

9. Solve an Euler equation:
$$x^2 y'' - 5xy' + 9y = 0$$

10. Solve an Euler equation:
$$x^2 y'' + 3xy' - 4y = 0$$

11. Solve an Euler equation:
$$25x^2 y'' + 25xy' + y = 0$$

12. Solve a non homogeneous Euler equation:
$$2x^2 y'' + 5xy' + y = x^2 - x$$

13. Taylor series solution of IVP:
$$y'' + y^2 = 1$$
$$y(0) = 2, \quad y'(0) = 3$$

14. Solve the equation by 3 methods. (A) As a constant coefficient equation, (B) By reduction of order method, type no-y, (C) By reduction of order method, type no-x.
$$3y'' - 2y' = 0$$

15. Solve the nonlinear equation by reduction of order:
$$yy'' = (y')^2$$

16. A mass of 4 pounds is attached to a spring whose constant is 2 lb/ft. The medium offers a damping force that is numerically equal to the instantaneous velocity. The mass is initially released from a point 1 foot above the equilibrium position with a downward velocity of 8 ft/s.

    Determine the time at which the mass passes through the equilibrium position.

    Find the time at which the mass attains its extreme displacement from the equailibrium position. What is the position of mass at this instant..

17. Find the charge on the capacitor in an LRC-series circuit when
$$L = \frac{1}{2}h,\ R = 10\Omega,\ C = 0.01f,\ E = 150V$$
with initial condition
$$q(0) = 1C,\ i(0) = 0A.$$
What is the charge on the capacitor after a long time?

18. Find linearly dependence

    (1) $\mathbf{u}_1 = \langle 0,1,1\rangle,\ \mathbf{u}_2 = \langle 1,2,0\rangle,\ \mathbf{u}_3 = \langle -1,0,2\rangle,$

    (2) $\mathbf{u}_1 = x + x^2,\ \mathbf{u}_2 = 1 + 2x,\ \mathbf{u}_3 = -1 + 2x^2,$

19. Given three vectors:
$$\mathbf{u}_1 = \langle 0,1,0\rangle,\ \mathbf{u}_2 = \langle 1,2,0\rangle,\ \mathbf{u}_3 = \langle 1,1,3\rangle,$$

    (a) Show linearly independence

    (b) Find a linear combination for $\mathbf{a} = \langle 0,1,0\rangle$, i.e, coordinates of $\mathbf{a}$, under the basis, $\mathbf{u}_1, \ldots.$

    (c) Find the orthgonal bases by the Gram-Schmidt orthogonalization process, $\mathbf{v}_1$ ....

    (d) Find the orthnormal bases by the Gram-Schmidt orthogonalization process, $\mathbf{w}_1$ ...

    (e) Find a linear combination for $\mathbf{a} = \langle 0,1,0\rangle$, i.e, coordinates of $\mathbf{a}$, under the orthnormal basis above $\mathbf{w}_1$ ....