M351 Study Guide 1 (S. Zhang) .

1. Verify that the piecewise defined function

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

is a solution of the differential equation

$$xy' - 2y = 0 \quad \text{on } (-\infty, \infty).$$

2. Solve IVP with the general solution $y = C_1 e^x + C_2 e^{-x}$:

$$y'' - y = 0, y(0) = 1, y'(0) = 2$$

3. Without solving the equation, find and plot the region in xy plane for the following equation such that there is a unique solution through each given initial point in this region

$$y' = \sqrt{xy}$$

4. Make a table for y' and plot the direction field, including at least points $(0,1)$, $(1,1)$ and $(1,0)$. Then sketch the solution curve corresponding to the given initial condition. Finally, use this solution curve to estimate the desired value of the solution:

$$\frac{dy}{dx} = y - x; \quad y(0) = 1, \quad y(3) = ?$$

5. Construct a table for y', y'' and solution curve $y(x)$ shapes. Find and classify critical points Sketch phase portrait (phase line + direction field). Sketchall typical solution curves on the graph of direction field.

$$y' = 4y - y^2.$$

6. Solve the equation and determine the interval in which the solution is defined.

$$y' = (1 - 2x)y^2, \ y(0) = -1/6$$

7. Solve the IVP. Determine an interval for which the initial value problem is certain to have a unique solution.

$$y' = \frac{2t}{2 - 2y}, \qquad y(3) = 2.$$

8. Solve IVP. Find the largest interval over which the solution is defined.

$$y\frac{dx}{dy} - x = 2y^2, \ y(1) = 5$$

9. Solve

$$y' + y = \begin{cases} 1 & x < 1 \\ -1 & x \geq 1 \end{cases} \quad y(0) = 1$$

10. Determine if the differential equation is exact, and solve it if it is.

$$(2y - \frac{1}{x} + \cos 3x)\frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y \sin 3x = 0$$

11. Solve the IVP:

$$(\frac{1}{1 + y^2} + \cos x - 2xy)\frac{dy}{dx} = y(y + \sin x), \ y(0) = 1$$

12. Solve the equation of $y' = f(y/x)$ form:

$$\frac{dy}{dx} = \frac{x + 3y}{3x + y}$$

13. Solve the equation of $y' = f(Ax + By + C)$ form:

$$y' = \frac{1 - x - y}{x + y}$$

14. Approximate $y(1.2)$ by the Euler method with $h = 0.1$ and $h = 0.05$ (using 4-decimal approximation):

$$y' = 2y + 2t + 1, \quad y(1) = 0$$

Compare the numerical solutions with exact solution listing errors and relative errors.

15. A 4-lb roast, initially at 50^0F, is placed in a 375^0F oven at 5:00PM. After 75 minutes it is found that the temperature $T(t)$ of the roast is 125^0F. When will the roast be 150^0F (medium rare)?

16. A tank contains 100 liters of salt water with a concentration of 2 g/liter. Pure water flows in at a rate of 2 liters/min, the well-stirred solution flowing out at the same rate. Find the time when the tank reaches 1 g/liter salt concentration.

17. The time rate of change of an alligator population P in a swamp is proportional to the square of P. The swamp contained a dozen alligators in 1988, two dozen in 1998. When will there be four dozen alligators in the swamp? What happens there after?

18. Find the linear dependence by both finding nonzero solutions and by Wrongskian:

$$y_1 = 1 + x^2, \ y_2 = x^2 - x, \ y_3 = x^2 + x$$

19. Given one solution, find another linear independent solution and the general solution:

$$x^2 y'' - y = 0 \quad y_1 = x^3$$

20. Find the roots of characteristic equation and the general solution:

(1) $4y'' - 4y' + y = 0$

(2) $(D-1)^2(D^2-1)^2((D-2)^2-9)((D-1)^2+4)^2 y = 0$