M351 H9 (S. Zhang) 8.3.

1. 8.3: 2-8, 11-13

1. (8.3:8) Find the rank by reducing the matrix to a row-echelon form.

$$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{pmatrix}$$

- **ans:**

Find a row-echelon form.

$$\begin{pmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{pmatrix} \xrightarrow{-r_1+r_2,-2r_1+r_3} \begin{pmatrix} 1 & -2 & 3 & 4 \\ & 6 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ & 9 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{-9r_3+r_4,-6r_3+r_2} \begin{pmatrix} 1 & -2 & 3 & 4 \\ & & 3 & 4 \\ 0 & 1 & 0 & 0 \\ & & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{r_3\leftrightarrow r_2} \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ & & 3 & 4 \\ & & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{(1/3)r_3} \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ & & 1 & 4/3 \\ & & 0 & 0 \end{pmatrix}$$

This is a row-echelon form. Rank is the number of nonzero rows,

$$Rank(A) = 3$$

2. (8.3:13) Determine linear dependence by the rank of corresonding matrix

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ -1 \\ 3 \\ -1 \end{pmatrix},\ \mathbf{u}_2 = \begin{pmatrix} 1 \\ -1 \\ 4 \\ 2 \end{pmatrix},\ \mathbf{u}_3 = \begin{pmatrix} 1 \\ -1 \\ 5 \\ 7 \end{pmatrix}$$

- **ans:** Here we can write A using $\mathbf{u}_i$ as row vectors or as column vectors.

Similar to the earlier method of finding nonzero solutions of

$$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3 = \mathbf{0}$$

we would like to write A using $\mathbf{u}_i$ as column vectors.

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 3 & 4 & 5 \\ -1 & 2 & 7 \end{pmatrix} \xrightarrow{r_1+r_2,-3r_1+r_3,r_1+r_4} \begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & 1 & 2 \\ & 3 & 8 \end{pmatrix}$$

$$\xrightarrow{-3r_3+r_4} \begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & 1 & 2 \\ & & 2 \end{pmatrix}$$

$$\xrightarrow{r_2\leftrightarrow r_3} \begin{pmatrix} 1 & 1 & 1 \\ & 1 & 2 \\ & 0 & 0 \\ & & 2 \end{pmatrix}$$

$$\xrightarrow{r_3\leftrightarrow r_4} \begin{pmatrix} 1 & 1 & 1 \\ & 1 & 2 \\ & & 2 \\ & & 0 \end{pmatrix}$$

$$\xrightarrow{(1/2)r_3} \begin{pmatrix} 1 & 1 & 1 \\ & 1 & 2 \\ & & 1 \\ & & 0 \end{pmatrix}$$

This is a a row-echelon form. Rank is the number of nonzero rows, 3. As the rank is the same as the number of vectors, the vectors are linearly independent.