M351 H8 (S. Zhang) 8.1-2.

1. 8.1: 11-12, 16-18,21,23,26-30, 35,38

1. (8.1:18) Find AB and BA

$$A = \begin{pmatrix} 1 & 4 \\ 5 & 10 \\ 8 & 12 \end{pmatrix},\ B = \begin{pmatrix} -4 & 6 & -3 \\ 1 & -3 & 2 \end{pmatrix}$$

- **ans:**

$$\begin{aligned} AB &= \begin{pmatrix} 1 & 4 \\ 5 & 10 \\ 8 & 12 \end{pmatrix} \begin{pmatrix} -4 & 6 & -3 \\ 1 & -3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -6 & 5 \\ -10 & 0 & 5 \\ -20 & 12 & 0 \end{pmatrix} \end{aligned}$$

But

$$\begin{aligned} BA &= \begin{pmatrix} -4 & 6 & -3 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 5 & 10 \\ 8 & 12 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 8 \\ 2 & -2 \end{pmatrix} \end{aligned}$$

So $AB \neq BA$ in genneral.

2. (8.1:38) Show that $AC = BC$, $C \neq 0$ does not necessarily imply $A = B$ by the example:

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix},\ B = \begin{pmatrix} 5 & 1 & 6 \\ 9 & 2 & -3 \\ -1 & 3 & 7 \end{pmatrix},$$
$$C = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

- **ans:**

$$\begin{aligned} AC &= \begin{pmatrix} 2 & 1 & 4 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 0 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{pmatrix} \\ BC &= \begin{pmatrix} 5 & 1 & 6 \\ 9 & 2 & -3 \\ -1 & 3 & 7 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 0 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{pmatrix} = AC \end{aligned}$$

But $A \neq B$.

3. 8.2: 3-4,8-10,17-18,29-30

1. (8.2:3) Solve the linear system by
(1) Gaussian elimination
(2) Gauss-Jordan elimination

$$\begin{aligned} 9x_1 \quad +3x_2 &= \quad -5 \\ 2x_1 \quad +x_2 &= \quad -1 \end{aligned}$$

- **ans:** (1) Row-echelon form:

$$\begin{pmatrix} 9 & 3 & | & -5 \\ 2 & 1 & | & -1 \end{pmatrix} \overset{(-2/9)r_1+r_2}{\to} \begin{pmatrix} 9 & 3 & | & -5 \\ & 1/3 & | & 1/9 \end{pmatrix}$$
$$\overset{(1/9)r_1}{\to} \begin{pmatrix} 1 & 1/3 & | & -5/9 \\ & 1/3 & | & 1/9 \end{pmatrix}$$
$$\overset{3r_2}{\to} \begin{pmatrix} 1 & 1/3 & | & -5/9 \\ & 1 & | & 1/3 \end{pmatrix}$$

Now backward substitution.

$$\begin{aligned} &x_2 = 1/3, \\ &x_1 = -5/9 - (1/3)x_2 = -2/3 \\ &x = \begin{pmatrix} -2/3 \\ 1/3 \end{pmatrix} \end{aligned}$$

(2) Reduced row-echelon form:

$$\begin{pmatrix} 9 & 3 & | & -5 \\ 2 & 1 & | & -1 \end{pmatrix} \overset{(-2/9)r_1+r_2}{\to} \begin{pmatrix} 9 & 3 & | & -5 \\ & 1/3 & | & 1/9 \end{pmatrix}$$
$$\overset{(1/9)r_1}{\to} \begin{pmatrix} 1 & 1/3 & | & -5/9 \\ & 1/3 & | & 1/9 \end{pmatrix}$$
$$\overset{3r_2}{\to} \begin{pmatrix} 1 & 1/3 & | & -5/9 \\ & 1 & | & 1/3 \end{pmatrix}$$
$$\overset{(-1/3)r_2+r1}{\to} \begin{pmatrix} 1 & & | & -2/3 \\ & 1 & | & 1/3 \end{pmatrix}$$

That is

$$x = \begin{pmatrix} -2/3 \\ 1/3 \end{pmatrix}$$

2. (8.2:8) Solve the linear system by
(1) Gaussian elimination
(2) Gauss-Jordan elimination

$$\begin{aligned} x_1 \quad +2x_2 \quad -4x_3 &= \quad 9 \\ 5x_1 \quad -x_2 \quad +2x_3 &= \quad 1 \end{aligned}$$

- **ans:** (1) Row-echelon form:

$$\begin{pmatrix} 1 & 2 & -4 & | & 9 \\ 5 & -1 & 2 & | & 1 \end{pmatrix} \overset{(-5)r_1+r_2}{\to} \begin{pmatrix} 1 & 2 & -4 & | & 9 \\ & -11 & 22 & | & -44 \end{pmatrix}$$
$$\overset{(-1/11)r_2}{\to} \begin{pmatrix} 1 & 2 & -4 & | & 9 \\ & 1 & -2 & | & 4 \end{pmatrix}$$

Now backward substitution. Note that x_3 is free. Let $x_3 = t$.

$$\begin{aligned} x_2 &= 4 + 2t, \\ x_1 &= 9 - 2x_2 + 4x_3 = 1 \\ x &= \begin{pmatrix} 1 \\ 4+2t \\ t \end{pmatrix} \\ &= t\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} \\ &= x_H + x_P \end{aligned}$$

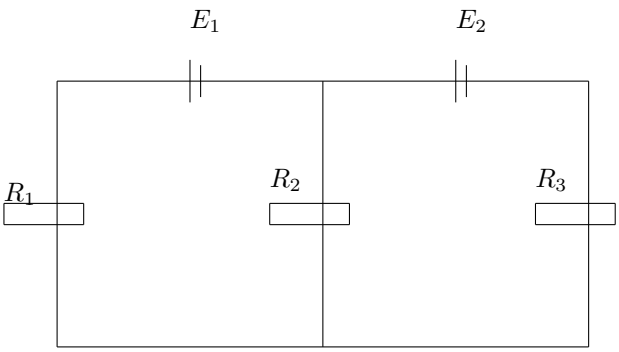
(2) Reduced row-echelon form:

$$\begin{pmatrix} 1 & 2 & -4 & | & 9 \\ 5 & -1 & 2 & | & 1 \end{pmatrix} \overset{(-5)r_1+r_2}{\to} \begin{pmatrix} 1 & 2 & -4 & | & 9 \\ & -11 & 22 & | & -44 \end{pmatrix}$$
$$\overset{(-1/11)r_2}{\to} \begin{pmatrix} 1 & 2 & -4 & | & 9 \\ & 1 & -2 & | & 4 \end{pmatrix}$$
$$\overset{(-2)r_2+r1}{\to} \begin{pmatrix} 1 & & & | & 1 \\ & 1 & -2 & | & 4 \end{pmatrix}$$

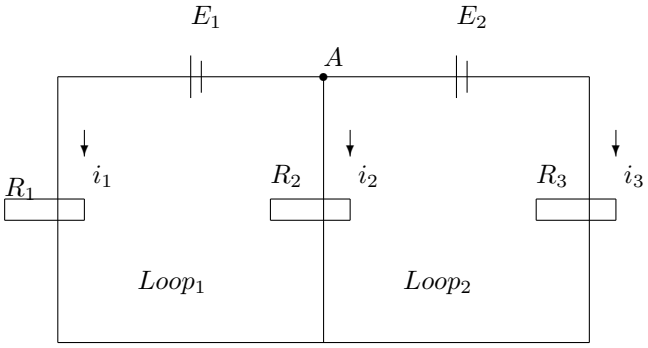
Note that x_3 is free. Let $x_3 = t$.

$$\begin{aligned} x_2 &= 4 + 2t, \\ x_1 &= 1 \\ x &= \begin{pmatrix} 1 \\ 4+2t \\ t \end{pmatrix} \\ &= t\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} \\ &= x_H + x_P \end{aligned}$$

3. Currents in a network. $R_1 = 3$ohms, $R_2 = 5$, $R_3 = 6$, $E_1 = 10$volts, $E_2 = 27$volts, . Find currents.



- **ans:** Define unkowns i_1, i_2, i_3,



Point rule: sum of currents is 0

Loop rule: sum of voltage drops is 0

$$\begin{aligned} -i_1 - i_2 - i_3 &= 0 && \text{(At point } A\text{)} \\ R_1 i_1 - R_2 i_2 &= E_1 && (\ Loop_1) \\ R_2 i_2 - R_3 i_3 &= E_2 && (\ Loop_2) \end{aligned}$$

We reduce the matrix form $(A|b)$ of the system to its rref:

$$\begin{pmatrix} -1 & -1 & -1 & | & 0 \\ 3 & -5 & 0 & | & 10 \\ & 5 & -6 & | & 27 \end{pmatrix} \overset{3r_1+r_2}{\to} \begin{pmatrix} -1 & -1 & -1 & | & 0 \\ & -8 & -3 & | & 10 \\ & 5 & -6 & | & 27 \end{pmatrix}$$
$$\overset{(-1)r_1}{\to} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ & -8 & -3 & | & 10 \\ & 5 & -6 & | & 27 \end{pmatrix}$$
$$\overset{(-1/8)r_2}{\to} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ & 1 & 3/8 & | & -5/4 \\ & 5 & -6 & | & 27 \end{pmatrix}$$
$$\overset{(-5)r_2+r_3}{\to} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ & 1 & 3/8 & | & -5/4 \\ & & -63/8 & | & 133/4 \end{pmatrix}$$
$$\overset{(-8/63)r_2+r_3}{\to} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ & 1 & 3/8 & | & -5/4 \\ & & 1 & | & -38/9 \end{pmatrix}$$
$$\overset{(-3/8)r_3+r_2}{\to} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ & 1 & & | & 1/3 \\ & & 1 & | & -38/9 \end{pmatrix}$$
$$\overset{(-1)r_3+r_1}{\to} \begin{pmatrix} 1 & 1 & & | & 38/9 \\ & 1 & & | & 1/3 \\ & & 1 & | & -38/9 \end{pmatrix}$$
$$\overset{(-1)r_2+r_1}{\to} \begin{pmatrix} 1 & & & | & 35/9 \\ & 1 & & | & 1/3 \\ & & 1 & | & -38/9 \end{pmatrix}$$

$$x = \begin{pmatrix} 35/9 \\ 1/3 \\ -38/9 \end{pmatrix}$$