M351 H7 (S. Zhang) vector.

1. 7.2: 51-52 7.3: 39-40 7.6: 23-28 7.7: 9-10

1. (7.7:a1)

$$\mathbf{a} = \langle 1, 1, -1\rangle, \ \mathbf{b} = \langle 1, 2, -2\rangle,$$

(a) Find a unit vector $\mathbf{c}$ parallel to $\mathbf{a}$.

(b) Find a vector 4 times long of $\mathbf{a}$, but in the opposite direction.

(c) Find $\mathbf{a} - 3\mathbf{b}$

(d) Find $\mathbf{a} \cdot \mathbf{b}$

(e) Find $proj_b\mathbf{a}$

(f) Write $\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2$ where $\mathbf{a}_1 \parallel \mathbf{b}$ and $\mathbf{a}_2 \perp \mathbf{b}$, show $\mathbf{a}_2 \cdot \mathbf{b} = 0$.

(g) Show $\mathbf{a}$ and $\mathbf{b}$ are linearly independent.

- **ans:**

(a) Find a unit vector $\mathbf{c}$ parallel to $\mathbf{a}$.

$$\mathbf{c} = \frac{\mathbf{a}}{\|\mathbf{a}\|} = \frac{1}{3}\langle 1, 1, -1\rangle$$

(b) Find a vector 4 times long of $\mathbf{a}$, but in the opposite direction.

$$\mathbf{c} = (4\|a\|)(-\frac{\mathbf{a}}{\|\mathbf{a}\|}) = -4\mathbf{a} = \langle -4, -4, 4\rangle$$

(c) Find $\mathbf{a} - 3\mathbf{b}$

$$\mathbf{a} - 3\mathbf{b} = \langle -2, -5, 5\rangle$$

(d) Find $\mathbf{a} \cdot \mathbf{b}$

$$\mathbf{a} \cdot \mathbf{b} = 1 + 2 + 2 = 5$$

(e) Find $proj_b\mathbf{a}$

$$proj_b\mathbf{a} = \frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b} = \frac{5}{9}1, 2, -2$$

By the way,

$$proj_a\mathbf{b} = (\mathbf{a} \cdot \frac{\mathbf{a}}{\|\mathbf{a}\|})\frac{\mathbf{a}}{\|\mathbf{a}\|} = \frac{\mathbf{b}\cdot\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}}\mathbf{a} = \frac{5}{3}\langle 1, 1, -1\rangle$$

(f) Write $\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2$ where $\mathbf{a}_1 \parallel \mathbf{b}$ and $\mathbf{a}_2 \perp \mathbf{b}$, show $\mathbf{a}_2 \cdot \mathbf{b} = 0$.

$$\mathbf{a}_1 = proj_b\mathbf{a} = \frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b} = \frac{5}{9}1, 2, -2$$

$$\mathbf{a}_2 = \mathbf{a} - \mathbf{a}_1 = \frac{1}{9}\langle 4, -1, 1\rangle$$

Checking:

$$\begin{aligned}\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2 &= \frac{1}{9}5 + 4, 10 - 1, -10 + 1, \ \textit{yes}\\ \mathbf{a}_1 &= \frac{5}{9}\mathbf{b}, \ \Rightarrow \ \mathbf{a}_1 \parallel \mathbf{b}\\ \mathbf{a}_2 \cdot \mathbf{b} &= \frac{1}{9}(4 - 2 - 2) = 0, \ \Rightarrow \ \mathbf{a}_1 \perp \mathbf{b}.\end{aligned}$$

(g) Show $\mathbf{a}$ and $\mathbf{b}$ are linearly independent.

Show there is no non zero solutions:

$$c_1\mathbf{a} + c_2\mathbf{b} = \mathbf{0}$$

$$\begin{aligned} c_1 \quad +c_2 &= 0\\ c_1 \quad +2c_2 &= 0\\ -c_1 \quad -2c_2 &= 0\end{aligned}$$

$c_1 = c_2 = 0$.

2. (7.7:a2)

$$\mathbf{u}_1 = \langle 1, 1, 1\rangle, \ \mathbf{u}_2 = \langle 0, 1, 1\rangle, \ \mathbf{u}_3 = \langle 1, 1, 0\rangle,$$

(a) Show linearly independence

(b) Find a linear combination for $\mathbf{a} = \langle 0, 1, 0\rangle$, i.e, coordinates of $\mathbf{a}$ under the basis.

(c) Find the orthgonal bases by Gram-Schmidt orthogonalization process

(d) Find the orthnormal bases by Gram-Schmidt orthogonalization process.

(e) Find a linear combination for $\mathbf{a} = \langle 0, 1, 0\rangle$, i.e, coordinates of $\mathbf{a}$ under the orthnormal basis.

- **ans:**

(a) Only zero solutions:

$$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3 = \mathbf{0}$$

$$\begin{aligned} c_1 \quad\quad +c_3 &= 0\\ c_1 \quad +c_2 \quad +c_3 &= 0\\ c_1 \quad +c_2 \quad\quad &= 0\end{aligned}$$

$c_1 = c_2 = c_3 = 0$.

(b) Solve linear system:

$$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3 = \mathbf{a}$$

$$\begin{aligned} c_1 \quad\quad +c_3 &= 0\\ c_1 \quad +c_2 \quad +c_3 &= 1\\ c_1 \quad +c_2 \quad\quad &= 0\end{aligned}$$

to get

$$c_1 = -1,\ c_2 = 1,\ c_3 = 1$$

$$\langle 0, 1, 0\rangle = (-1)\mathbf{u}_1 + (1)\mathbf{u}_2 + (1)\mathbf{u}_3$$

Coordinates $(-1, 1, 1)$.

(c) Orthogonal basis.

$$\mathbf{v}_1 = \mathbf{u}_1 = \langle 1, 1, 1\rangle$$
$$\mathbf{v}_2 = \mathbf{u}_2 - proj_{\mathbf{v}_1}\mathbf{u}_1 = \langle -\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\rangle$$
$$\mathbf{v}_3 = \mathbf{u}_3 - proj_{\mathbf{v}_1}\mathbf{u}_3 - proj_{\mathbf{v}_2}\mathbf{u}_3 = \langle 0, \frac{1}{2}, -\frac{1}{2}\rangle$$

(d) Orthonormal basis.

$$\mathbf{w}_i = \mathbf{v}_i/\|\mathbf{v}_i\|$$

$$\mathbf{w}_1 = \frac{1}{\sqrt{3}}\langle 1, 1, 1\rangle$$
$$\mathbf{w}_2 = \frac{1}{\sqrt{6}}\langle -2, 1, 1\rangle$$
$$\mathbf{w}_3 = \frac{1}{\sqrt{2}}\langle 0, 1, -1\rangle$$

(e) The coordinates under $\mathbf{w}_i$.
2 method:
(1) solving lienar system as in (b):

$$c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3 = \mathbf{a}$$

(very difficult) to get

$$c_1 = 0.5774,\ c_2 = 0.4082,\ c_3 = 0.7071$$

Coordinates $(0.5774, 0.4082, 0.7071)$.
(2) using inner products.

$$c_1 = \mathbf{a}\cdot\mathbf{w}_1 = \frac{1}{\sqrt{3}}$$

$$c_2 = \mathbf{a}\cdot\mathbf{w}_2 = \frac{1}{\sqrt{6}}$$

$$c_3 = \mathbf{a}\cdot\mathbf{w}_3 = \frac{1}{\sqrt{2}}$$

$$\mathbf{a} = \frac{1}{\sqrt{3}}\mathbf{w}_1 + \frac{1}{\sqrt{6}}\mathbf{w}_2 + \frac{1}{\sqrt{2}}\mathbf{w}_3$$

Coordinates $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}) = (0.5774, 0.4082, 0.7071)$.