M351 H3 2.6-8 (S. Zhang) .

1. 2.6: 2-4

10.99

1. (2.6:2) Approximate $y(0.2)$ by the Euler method with
10.26 $h = 0.1$ and again with $h = 0.05$:

$$y' = x + y^2, \quad y(0) = 0$$

- **ans:**

$$t_i = t_0 + ih,\ y_{i+1} = y_i + hf(t_i, y_i)$$

$$t_0 = 0,\ h = 0.1,\ y_0 = 0$$

$$y_1 = y_0 + hf(t_0, y_0) = 0 + 0.1(0) = 0$$

$$t_1 = 0.1$$

$$y_2 = y_1 + hf(t_1, y_1) = 0 + 0.1(0.1) = 0.01$$

Again, with with $h = 0.05$:

$$t_i = t_0 + ih,\ y_{i+1} = y_i + hf(t_i, y_i)$$

$$t_0 = 0,\ h = 0.05,\ y_0 = 0$$

$$y_1 = y_0 + hf(t_0, y_0) = 0 + 0.05(0) = 0$$

$$t_1 = 0.05$$

$$y_2 = y_1 + hf(t_1, y_1) = 0 + 0.05(0.05) = 0.0025$$

$$t_2 = 0.1$$

$$y_3 = y_2 + hf(t_2, y_2) = 0.0075$$

$$t_3 = 0.15$$

$$y_4 = y_3 + hf(t_3, y_3) = 0.015003$$

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1. 2.7: 3-4, 13-14, 21-23

810.99

1. (2.7:13) A thermometer is removed from a room where
810.31 the temprature is 70 degrees. And is taken outside where the air termperature is 10 degrees. After one-half minute the thermometer reads 50 degrees. What is the reading of the thermometer at 1 min? How long will it take for the termometer to reach 15 degrees?

- **ans:**

$$\begin{aligned} T(0) &= 70 \\ A &= 10 \\ T(0.5) &= 50 \\ \frac{dT}{dt} &= k(A - T) = k(10 - T) \\ \int \frac{dT}{10 - T} &= \int kdt \\ -\ln(10 - T) &= kt + C, \\ T &= 10 - Be^{-kt} \end{aligned}$$

$T(0) = 70$, $T(0.5) = 50$

$$\begin{aligned} T &= 10 + 60e^{-kt} \\ 50 &= 10 + 60e^{-k/2} \\ k &= (-2)\log(40/60) = 0.8109 \\ T &= 10 + 60e^{-0.8109t} \\ T(1) &= 36.337 \end{aligned}$$

Find t:

$$\begin{aligned} 15 &= 10 + 60e^{-0.8109t} \\ t &= \log(5/60)/(-0.8109) = 3.06(min) \end{aligned}$$

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1. 2.8: 2-3, 11-12, a1-a2

811.99

1. (2.8:a1) Suppose that when a certain lake is stocked
811.11 with fish, the birth and death rates β and δ are both inversely proportional to $\sqrt{P}$. Show that

$$P(t) = \left(\frac{1}{2}kt + \sqrt{P_0}\right)^2$$

If $P_0 = 100$ and after 6 months, there are 169 fish in the lake, how many will there be after 1 year?

- **ans:** Here we have an extra hint as the solution is given!

Note the population is proportional to the size of the population. Here proportion rate is: (birth rate $\beta = k_1/\sqrt{P}$ and death rate $\delta = k_2/\sqrt{P}$)

$$c = \beta - \delta = k/\sqrt{P}.$$

So

$$\frac{d}{dt}P = cP = k\frac{1}{\sqrt{P}}P = k\sqrt{P}.$$

$$\int \frac{1}{\sqrt{P}}dP = kdt$$

$$2\sqrt{P} = kt + C$$

$P(0) = P_0$:

$$2\sqrt{P_0} = 0 + C$$

So we prove that

$$P(t) = \left(\frac{1}{2}kt + \sqrt{P_0}\right)^2$$

Further, $P(0) = 100$, $P(6) = 169$, then $k = 1$, and

$$P(12) = (\frac{t}{2} + 10)^2 = 256$$
