M351 Tutorial/Lab 2 (S. Zhang) .

1. Find r so that $y = t^r$ is a solution

$$2t^3y'' - t^2y' + ty = 0.$$

Solution:

```
restart; y:=t^r;
eq:=2*t^3*diff(y,t$2)-t^2*diff(y,t)-t*y=0;
fsolve(eq,r);
fsolve(subs(t=1,eq),r);

answer   -.2807764064, 1.780776406
```

2. (300) Find r so that $y = t^r$ is a solution

$$2ty' - \pi y = 0.$$

M351 Tutorial/Lab 3 (S. Zhang) .

1. Find $\mu(3)$ where $\mu(t)$ is the integrating factor for the first order linear equation

$$4y' + ty = 12t^2$$

Solution: Rewrite the equation in the standard form:

$$y' + \frac{t}{4}y = 3t^2$$

```
mu:=exp(int(t/4, t));
subs(t=3,mu); # no need
evalf(subs(t=3,mu));
3.080216849
```

2. (276) Find $\mu(2)$ where $\mu(t)$ is the integrating factor for the first order linear equation

$$ty' - 3y = 12t^2$$

M351 Tutorial/Lab 4 (S. Zhang) .

1. Find $y(2)$ for the first order linear equation

$$ty' + 3y = 12t^2,\ y(1) = 2$$

Solution:

```
dsolve({t*diff(y(t),t)+3*y(t)=12*t^2,
  y(1)=2}, y(t));
evalf(subs(t=2,rhs(%)));
answer  9.550000000
```

2. (277) Find $y(4)$ for the first order linear equation

$$ty' - y = \ln(3t),\ y(2) = 3$$

M351 Tutorial/Lab 5 (S. Zhang) .

1. Find y_0 so that the solution passes the point $(3, 4)$ for the first order linear equation

$$y' - 3y = 12,\ y(1) = y_0$$

Solution:

```
dsolve({ diff(y(t),t)-3*y(t)=12,
  y(1)=c}, y(t));
fsolve(subs(t=3,rhs(%))=4,c);
answer
  -3.980169982
```

2. (278) Find y_0 so that the solution passes the point $(2, 4)$ for the first order linear equation

$$y' + ty = 12t^2,\ y(1) = y_0$$

M351 Tutorial/Lab 6 (S. Zhang) .

1. Show the equation is exact

$$(3x^2y - 2x)dx + (\cos y + x^3)dy = 0$$

Find $M_y(4, 3)$ and $N_x(4, 3)$ in decimal.

Solution:

```
M:=3*x^2*y-2*x; N:=cos(y)+x^3;
is(diff(M,y)=diff(N,x));
evalf(subs({x=4,y=3},diff(M,y)));
evalf(subs({x=4,y=3},diff(N,x)));

answer 48
```

2. (281) Show the equation is exact

$$(3x^2 \sin y - 2x)dx + (e^y + x^3 \cos y)dy = 0$$

Find $M_y(1, -3)$ and $N_x(1, -3)$ in decimal.

M351 Tutorial/Lab 7 (S. Zhang) .

1. Find y_0 so that the $\lim_{t\to\infty} y(t)$ is finite.

$$y' - \pi y = e^{-7t},\ y(2) = y_0.$$

Solution:

```
dsolve({diff(y(t),t)-Pi*y(t)
=exp(-7*t),y(2)=a}, y(t));
k:=limit(rhs(%),t=infinity);
fsolve( (7*a+Pi*a+exp(-14)) = 0, a)
% copy and paste from k output!

answer  -8.199192649 10^(-8)
```

2. (299) Find y_0 so that the $\lim_{t\to\infty} y(t)$ is finite.

$$y' - 1.2y = e^{-3.1t},\ y(0) = y_0.$$

M351 Tutorial/Lab 8 (S. Zhang) .

1. Approximate $y(1.2)$ by the Euler method with $h = 0.1$:

$$y' = 2y^2 + 2t + 1, \quad y(1) = 0$$

Solution:

$$y' = f(t, y) = 2y^2 + 2t + 1$$

$$t_i = t_0 + ih,\ y_{i+1} = y_i + hf(t_i, y_i)$$

$$t_0 = 1,\ h = 0.1,\ y_0 = 0$$

```
f:=(T,Y)-> 2*Y^2+2*T+1;
h:=0.1; t:=1; y:=0;
y:=y+h*f(t,y); evalf(y);
t:=t+h; f(t,y);
y:=y+h*f(t,y); evalf(y);

ans 0.638
```

2. (282) Approximate $y(2.3)$ by the Euler method with $h =$ 0.1:

$$y' = 2y^2 + e^{2t} + 1, \quad y(2) = 3$$

M351 Tutorial 9 (S. Zhang) .

1. Find the roots of the characteristic equation for the DE

$$4y'' - 4y' - 10y = 0$$

Find the numerical values.
Solution:

$$4r^2 - 4r + 3 = 0$$

```
solve(4*r^2 - 4*r - 10, r);
evalf(%);

ans 2.158312395, -1.15831239
```

2. (283) Find the roots of the characteristic equation for the DE

$$y'' + 4y' - 7y = 0$$

Find the numerical values.

M351 Tutorial 10(a) (S. Zhang) .

1. Show linear dependence by Wronskian:

$$f(x) = \sin^2 x; \quad g(x) = 1 - \cos 2x;$$

Solution:

```
f:=sin(x)^2; g:=1-cos(2*x);
with(linalg);
Wm:=matrix(2,2,[f,g,diff(f,x),diff(g,x)]);
W:=det(Wm);
simplify(W)

ans 0
```

2. Show linear independence by Wronskian and evaluate $W(2)$:

$$f(x) = \sin x; \quad g(x) = 1 - \cos 2x;$$

Solution:

```
f:=sin(x); g:=1-cos(2*x);
with(linalg);
Wm:=matrix(2,2,[f,g,diff(f,x),diff(g,x)]);
W:=det(Wm);
W:=simplify(W);
evalf(subs(x=2,W));

ans -.6881585
```

3. (285) Show linear independence by Wronskian and evaluate $W(3)$:

$$f(x) = \sin x; \quad g(x) = e^{2x} \cos x;$$
