M351 Tutorial/Lab 11(b) (S. Zhang) .

1. Find y_p by the method of undetermined coefficients:

$$y'' + 2y' - \sqrt{5}y = t - 4.$$

Evaluate the determined coefficients.

Solution:

```
restart;
y:=t->A*t+B;
left:=(D(D(y))+2*D(y)-sqrt(5)*y)(t);
e:=left=t-4;
so:=solve({subs(t=0,e),subs(t=1,e)},{A,B});
evalf(so);

ans {B = 1.388854382, A = -.4472135954}
## last two steps can be combined:
so:=fsolve({subs(t=0,e),subs(t=1,e)},{A,B});
```

2. Find y_p by the method of undetermined coefficients:

$$y'' + 3y' - 6^{1/3}y = 3e^{-3t} + 1$$

Evaluate the determined coefficients.

M351 Tutorial/Lab 12(c) (S. Zhang) .

1. If $W(2) = 3$. Find $W(4)$ without solving the equation

$$t^2y'' - 2y' + (3 + t)y = 0$$

Solution: By hand:

$$y'' - \frac{2}{t^2}y' + \frac{3+t}{t^2}y = 0$$

$$W = ce^{-\int p} = ce^{-\int -2dt/t^2} = \frac{c^{-2/t}}{e}$$

$$W(2) = 3, \quad c = 3e, \; W(4) = 3ee^{-2/4} = 3\sqrt{e}$$

By Maple:

```
W:=exp(int(-2/t^2, t));
evalf(subs(t=4,W)*3/subs(t=2,W))
# ans 1.819591980
```

2. If $W(1) = 4$. Find $W(2)$ without solving the equation

$$t^3y'' - 4y' + ty = 0$$

M351 Tutorial/Lab 13(d) (S. Zhang) .

1. Given functions are linearly indepdent solutions to the corresponding homogeneous equation. Find the general solution by the method of variation of parameters.

$$t^2y'' - 2y = 3t^2 - 1, \; y_1 = t^2, \; y_2 = t^{-1}$$

Find $u_1'(4)$.

Solution: By hand: Plug $y_1 = t^2, \; y_2 = t^{-1}$ into the equation, and we can see they are solutions.

$$t^2y'' - 2y = 0$$

The Wronskian is

$$W(t^2, t^{-1}) = -3 \neq 0$$

They are linearly independent.

$$y_c = C_1t^2. + C_2t^{-1}$$

The UC does not work.

For the VP method,

$$y_p = u_1t^2. + u_2t^{-1}$$

$$u_1't^2 + u_2't^{-1} = 0$$
$$2u_1't - u_2't^{-2} = 3 - t^{-2}$$

$$u_1' = -u_2't^{-3}$$

$$-2u_2't^{-2} - u_2't^{-2} = 3 - t^{-2}$$

$$u_2' = -t^2 + 1/3$$

$$u_2 = -t^3/3 + t/3$$

$$u_1' = -u_2't^{-3} = 1/t - 1/(3t^3)$$

$$u_1 = \ln t + 1/6t^{-2}$$

$$y_p = u_1t^2. + u_2t^{-1} = t^2 \ln t + \frac{1}{2} - \frac{1}{3}t^2$$

$$y = y_c + y_p = C_1t^2. + C_2t^{-1} + t^2 \ln t + \frac{1}{2}$$

$$u_1' = 1/t - 1/(3t^3)$$

$$u_1'(4) = 1/4 - 1/(3(4^3)) = \frac{47}{3(4^3)}$$

```
 restart;
y1:=t^2; y2:=t^(-1);
s:=solve({u*y1+v*y2=0,
 u*diff(y1,t)+v*diff(y2,t)
 =3- t^(-2) },{u,v});
u1:=int(rhs(s[1]),t);
u2:=int(rhs(s[2]),t);
simplify(u1*y1+u2*y2);
evalf(subs(t=4,rhs(s[1])));
#ans 0.2447916667
evalf(subs(t=4,rhs(s[2])));
#ans -15.66666667
#(might be this, input both)
```

2. Given functions are linearly indepdent solutions to the corresponding homogeneous equation. Find the general solution by the method of variation of parameters.

$$y'' - 4y' = 3t^2 - 1,\ y_1 = e^{2t},\ y_2 = e^{-2t}$$

Find $u_1'(3)$.

M351 Tutorial/Lab 14(e) (S. Zhang) .

1. Write y in $R\cos(\omega t - \delta)$ form:

$$y = \cos 3t - 2\sin t$$

Solution: By hand:

$$\begin{aligned} y &= A\cos\omega t + B\sin\omega t \\ &= R\cos(\omega t - \delta) \\ R &= \sqrt{A^2 + B^2} = \sqrt{5} \\ \delta &= \tan^{-1}\frac{B}{A} = \tan^{-1}(-\frac{1}{2}) \end{aligned}$$

```
A:=1; B:=-2;
R:=sqrt(A^2+B^2);
del:=arctan(B/A);
evalf({R, del});
# ans 2.236067977, -1.107148718
```

2. Write y in $R\cos(\omega t - \delta)$ form:

$$y = 9\cos 4t - 5\sin 4t$$

M351 Tutorial/Lab 15(f) (S. Zhang) .

1. Find the Taylor series solution of the first 10 terms for

$$y'' + x^2 y' = 0, \quad y(0) = 1,\ y'(0) = 3$$

Find $y(1)$ of the above series solution.

Solution:

By hand,

$$\begin{aligned} y'' &= -x^2 y' & y''(0) &= -0^2(3) = 0 \\ y''' &= -2xy' - x^2 y'' & y'''(0) &= 0 \\ &\ldots \end{aligned}$$

By Maple build-in functions:

```
eq:=diff(y(x),x,x)+x^2*diff(y(x),x)=0;
sol:=dsolve({eq,y(0)=1,D(y)(0)=3}, y(x));
evalf(subs(x=4,rhs(sol)));
evalf(subs(x=1,rhs(sol)));
Order:=10;
soln:=dsolve({eq,y(0)=1,D(y)(0)=3},
 y(x),series);
sol:=convert(rhs(soln),polynom);
evalf(subs(x=4,sol));
evalf(subs(x=1,sol));

#ans 3.773809
```

2. Find the series solution of the first 10 terms for

$$y'' + xy' = 0, \quad y(0) = 2,\ y'(0) = 3$$

Find $y(1/3)$ of the above series solution.

M351 Tutorial/Lab 16(g) (S. Zhang) .

1. Find the determinant by both Maple and Matlab.

$$A = \begin{pmatrix} -\sqrt{3} & -1 \\ 4.2^{2.1} & -1 \end{pmatrix}$$

Solution:

Maple:

```
 with(linalg):
A:=matrix(2,2,[-sqrt(3),-1, 4.2^2.1,-1]);
d:=det(A);
determinant=evalf(d);

determinant = 22.09419506
```

Matlab:

```
  format long
  A=[-sqrt(3),-1;   4.2^2.1,-1]
  d=det(A)

ans = 22.094195061092300
```

Input both Maple and Matlab codes and output into the online submission window.

2. Find the determinant by both Maple and Matlab.

$$A = \begin{pmatrix} \sqrt{5} & -\sin 1 & 3 \\ e^2 & \pi & 2 \\ 0 & -\sin \pi 5 & -3.2 \end{pmatrix}$$

Note, in Maple and in Matlab $e = exp(1)$, but $\pi = Pi$ or $\pi = pi$.

M351 Tutorial/Lab 17(h) (S. Zhang) .

1. Matlab: Find the orthonomal basis by Gram-Scmidt:

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \ \mathbf{u}_2 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \ \mathbf{u}_3 = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$$

Solution:

Matlab:

```
  format short
  u1=[1 2 3]'
  u2=[2 4 1]'
  u3=[-1 3 1]'
  v1=u1; w1=u1/norm(u1,2)
  v2=u2-(u2'*w1)*w1;
  w2=v2/norm(v2,2)
  v3=u3-(u3'*w1)*w1-(u3'*w2)*w2;
  w3=v3/norm(v3,2)
  basisW= [w1 w2 w3]

% Method 2 -- improved Gram-Schmidt
%  Use Method 1 above for Lab.

  A=[u1 u2 u3]
  [q r]=qr(A);
   basisW= q

basisW =

    0.2673    0.3586   -0.8944
    0.5345    0.7171    0.4472
    0.8018   -0.5976   -0.0000
```

2. Matlab: Find the orthonomal basis by Gram-Scmidt:

$$\mathbf{u}_1 = \begin{pmatrix} \sqrt{3} \\ 2 \\ -3 \end{pmatrix}, \ \mathbf{u}_2 = \begin{pmatrix} 2 \\ e^4 \\ 1 \end{pmatrix}, \ \mathbf{u}_2 = \begin{pmatrix} -1 \\ 3 \\ 1.1^9 \end{pmatrix}$$

M351 Tutorial/Lab 18(i) (S. Zhang) .

1. Matlab: Find $2A^T - 3B$ and AB^TC:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \ B = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

Solution:

Matlab:

```
  A=[1  1; 1 2]
  B=[2  3; -1 2]
  C=[1 2 3; 3 2 1]
  ans1=2*A'-3*B
  ans2=A*B'*C

ans1 =

    -4    -7
     5    -2


ans2 =

     8    12    16
    17    22    27
```

2. Matlab: Find $3C^TA$ and $(A - B)C - AC + BC$:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \ B = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 2 & 3 \\ \sqrt{3} & \sqrt{2} & 1 \end{pmatrix}$$

M351 Tutorial/Lab 19(j) (S. Zhang) .

1. Matlab: Solve $Ax = b$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ -1 & 2 & -1 \end{pmatrix}, \ b = \begin{pmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \end{pmatrix}$$

Solution:

Matlab:

```
  A=[ 1 2 3; 3 2 1; -1 2 -1]
  b=[1 sqrt(2) sqrt(3)]'
  method1=A\b
  method2=inv(A)*b
  method3=rref([A b])

method1 =

    0.0161
    0.7785
   -0.1910
```

2. Matlab: Solve $Ax = b$

$$A = \begin{pmatrix} 1 & \sqrt{2} & 3 \\ 3 & \sqrt{2} & 1 \\ -1 & \sqrt{2} & -1 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ -\sqrt{2} \\ \sqrt{3} \end{pmatrix}$$

M351 Tutorial/Lab 20(k) (S. Zhang) .

1. Matlab: Find Rank(A) and the consistence of $Ax = b$ by the ranks. Then find one solution of $Ax = b$ if it is consistent

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 4 & 4 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Solution:

Matlab:

```
A=[ 1 2 3; 3 2 1; 4 4 4]
b=[1 2 3]'
rank(A)
rank(A)-rank([A b])
rref([A b])

Yes, the ranks are same, consisten.

Let x3=0, one solution is
 0.50
 0.25
 0.00
```

2. Matlab: Find Rank(A) and the consistence of $Ax = b$ by the ranks. Then find one solution of $Ax = b$ if it is consistent

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ -4 & -4 & -4 \end{pmatrix},\ b = \begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{pmatrix}$$

M351 Tutorial/Lab 21(l) (S. Zhang) .

1. Matlab: Find linear dependence by Rank(A). Then find one zero solution of $Ax = 0$ if it is.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 4 & 4 \end{pmatrix}$$

Solution:

Matlab:

```
A=[ 1 2 3; 3 2 1; 4 4 4]
 rank(A)
rref(A)

Yes, the rank is less than 3

Let x3=1, one solution is
 1
 -2
 1
```

2. Matlab: Find linear dependence by Rank(A). Then find one zero solution of $Ax = 0$ if it is.

$$A = \begin{pmatrix} 1 & \sqrt{2} & 3 \\ 3 & \sqrt{2} & 1 \\ 4 & 2\sqrt{2} & 4 \end{pmatrix}$$
