M242 Study Guide for E final (S. Zhang) .

1. Use Newton's method to find x_3:
$$x^5 - x - 1 = 0,\ x_1 = 1$$

2. Find the limit: $\lim_{x\to 0^+} \sin x \ln x$

3. Find the volume of the solid obtained by rotating the region bounded by the given cures about the specified line, by
(1) the method of rotation (adding washers/disks),
(2) the method of cylindrical shell (adding thin cylindrical shells).
Sketch the region, and the solid.
$$y = x,\ y = \sqrt{x},\ \text{ about } y = 1.$$

4. Find $\int e^{2y} \sin y dy$

5. Find $\int \sin^4 x dx.$

6. Find $\int \frac{3-x}{(x-1)(x^2-1)} dx$

7. Find $T_{h=2}, T_{h=1}, S_{h=1}$, if
$$\int_{\sin 8}^{2+\sin 8} f(x)\,dx,\ R_{h=2} = 0,\ L_{h=2} = 512;\ M_{h=2} = 32,$$

8. Find $\int_{-1}^{2} \frac{x}{\sqrt{|x-1|}} dx$ (hint: $\sqrt{|x-1|} = \sqrt{1-x}$ when $x \le 1$.)

9. Test if the following sequence or series converges or diverges and show your work (give, at least, the name of the test used). If a sequence or a series converges, find the limit or the sum.

(a) $2, \frac{3}{2}, \frac{4}{3}, \ldots, \frac{n+1}{n}, \ldots$

(b) $2 + \frac{3}{2} + \frac{4}{3} + \ldots + \frac{n+1}{n} + \ldots = \sum_{n=1}^{\infty} \frac{n+1}{n}$

(c) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \ldots, \frac{1}{n(n+1)}, \ldots$

(d) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

10. Write the periodic decimal as a fraction, $1.2\overline{345}$.

11. Determine absolute convergence, conditional convergence and divergence:
$$(1) \sum \left(\frac{3+2n}{2-n^2}\right)^n, \quad (2) \sum \frac{(-3)^n}{n^3}, \quad (3) \quad \sum \frac{2^n}{n!},$$
$$(4) a_1 = 2,\ a_{n+1} = \frac{2n+1}{n+10} a_n,\ \sum a_n.$$

12. Find the absolute convergence, conditional convergence and divergence by two methods: $\sum_{n=1}^{\infty} 2ne^{-n^2}$
(1) Root test or ratio test;
(2) Integral test or comparison test.

13. Find the radius and interval of convergence: $\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2n+1}$

14. Suppose $\sum_{n=0}^{\infty} c_n (x-1)^n$ converges for $x = 4$ and diverges for $x = -3$. What can be said about the convergence of divergence of
$$\sum_{n=0}^{\infty} c_n, \qquad \sum_{n=0}^{\infty} c_n 5^n, \qquad \sum_{n=0}^{\infty} c_n (-3)^n,$$

15. Find the power series (by the geometric series) and the Taylor series $f(x) = \ln(1+3x)$, $a = 0$ You need to find the first 3 nonzero terms.

16. Find the limit $\lim_{x\to 0} \frac{\sin x - x - x^2}{\cos x - 1}$ by both methods: (1) L'Hopital rule, (2) Taylor series expansion.

17. Find the Taylor polynomial $T_3(x)$ for $f(x) = \ln(1+2x)$, $a = 1$, $x \in [0.5, 1.5]$. And bound the error for $T_3(x)$ by R_3 on the given interval.

18. Given $\begin{cases} x = 1 + 3t^2 \\ y = 4 + 2t^3, \end{cases}$ $0 \le t \le 1$
(1) Find dy/dx and d^2y/dx^2 at $t = 1$.
(2) Find an equation of the line tangent at $t = 1$.
(3) Find the area of the region that lies under the curve.
(4) Find the arc length.

19. Fill:

$(x,\ y)$	(r, θ)
$(3, -3)$	$(\quad , \quad)$
$(\quad , 2)$	$(\quad , -\pi/2)$

20. (1)Sketch the curves (must show the data table for the points),
(2) find the intersection points (in (r, θ) coordinates), and
(3) find the area of the region inside both $r = 2 + 2\cos\theta$ and $r = 2$.

21. Find the equation for the parabola, and sketch:
vertex $(3, 2)$, focus $(3, 6)$

22. Find the equation of an ellipse, then sketch:
foci $(0, 2), (0, 6)$, vertices $(0, 0), (0, 8)$

23. Identify the type of conic section and find the vertices and foci, and sketch: $y^2 + 2y = 4x^2 + 3$