M242 Q12(c) (S. Zhang) (8 points). Name:______________

1. Find the power series (by the geometric series) and the Taylor series

$$f(x) = \ln(1+3x),\ a = 0$$

You need to find the first 3 nonzero terms.

- **ans:** To find the power series representation, we use only the formula for geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$$

$x \to -3x$

$$\frac{1}{1+3x} = 1 - 3x + 9x^2 - ...$$

Integrate both sides

$$\int \frac{dx}{1+3x} = \int (1 - 3x + 9x^2 - ...)dx$$
$$\frac{1}{3}\ln(1+3x) = x - \frac{3}{2}x^2 + 3x^3 - ... + C$$

Let $x = 0$ to find out C:

$$0 = \frac{1}{3}\ln(1+0) = 0 - 0 + 0 - ... + C$$

Therefore

$$\frac{1}{3}\ln(1+3x) = x - \frac{3}{2}x^2 + 3x^3 - ...$$
$$\ln(1+3x) = 3x - \frac{9}{2}x^2 + 9x^3 - ...$$

The second method – Taylor formula:

$$\begin{aligned} f(x) &= \ln(1+3x), & f(0) &= 0 \\ f'(x) &= 3(1+3x)^{-1}, & f'(0) &= 3 \\ f''(x) &= -9(1+3x)^{-2}, & f''(0) &= -2\cdot 9 \\ f'''(x) &= 2\cdot 27(1+3x)^{-3}, & f'''(0) &= 2\cdot 27 \end{aligned}$$

$$\begin{aligned} f(x) &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... \\ &+ \frac{f^{(n)}(a)}{n!}(x-a)^n + ... \\ \ln(1+3x) &= 0 + 3x + \frac{-9}{2!}x^2 + \frac{2\cdot 27}{3!}x^3 + ... \\ &= 3x - \frac{9}{2}x^2 + 9x^3 - ... \end{aligned}$$