

M242 Q12(b) (S. Zhang) (8 points). Name: _____

1. Find the power series (by the geometric series) and the Taylor series

$$f(x) = \ln(1 + 3x), \quad a = 0$$

You need to find the first 3 nonzero terms.

• **ans:** To find the power series representation, we use only the formula for geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$x \rightarrow -3x$$

$$\frac{1}{1+3x} = 1 - 3x + 9x^2 - \dots$$

Integrate both sides

$$\int \frac{dx}{1+3x} = \int (1 - 3x + 9x^2 - \dots) dx$$
$$\frac{1}{3} \ln(1+3x) = x - \frac{3}{2}x^2 + 3x^3 - \dots + C$$

Let $x = 0$ to find out C :

$$0 = \frac{1}{3} \ln(1+0) = 0 - 0 + 0 - \dots + C$$

Therefore

$$\frac{1}{3} \ln(1+3x) = x - \frac{3}{2}x^2 + 3x^3 - \dots$$
$$\ln(1+3x) = 3x - \frac{9}{2}x^2 + 9x^3 - \dots$$

The second method – Taylor formula:

$f(x) = \ln(1+3x),$	$f(0) = 0$
$f'(x) = 3(1+3x)^{-1},$	$f'(0) = 3$
$f''(x) = -9(1+3x)^{-2},$	$f''(0) = -2 \cdot 9$
$f'''(x) = 2 \cdot 27(1+3x)^{-3},$	$f'''(0) = 2 \cdot 27$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$
$$+ \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots$$
$$\ln(1+3x) = 0 + 3x + \frac{-9}{2!}x^2 + \frac{2 \cdot 27}{3!}x^3 + \dots$$
$$= 3x - \frac{9}{2}x^2 + 9x^3 - \dots$$