M242 Q12(a) (S. Zhang) (8 points). Name:______________

1. Find the power series (by the geometric series) and the Taylor series

$$f(x) = \ln(1 + 2x), \; a = 0$$

You need to find the first 4 nonzero terms.

- **ans:** To find the power series representation, we use only the formula for geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$$

$x \to -2x$

$$\frac{1}{1+2x} = 1 - 2x + 4x^2 - 8x^3 + ...$$

Integrate both sides

$$\int \frac{dx}{1+2x} = \int (1 - 2x + 4x^2 - 8x^3 + ...)dx$$
$$\frac{1}{2}\ln(1+2x) = x - x^2 + \frac{4}{3}x^3 - 2x^4 + ... + C$$

Let $x = 0$ to find out C:

$$0 = \frac{1}{2}\ln(1+0) = 0 - 0 + 0 - ... + C$$

Therefore

$$\frac{1}{2}\ln(1+2x) = x - x^2 + \frac{4}{3}x^3 - 2x^4 + ...$$
$$\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 - 4x^4 + ...$$

The second method – Taylor formula:

$$f(x) = \ln(1+2x), \qquad f(0) = 0$$
$$f'(x) = 2(1+2x)^{-1}, \qquad f'(0) = 2$$
$$f''(x) = -4(1+2x)^{-2}, \qquad f''(0) = -4$$
$$f'''(x) = 16(1+2x)^{-3}, \qquad f'''(0) = 16$$
$$f^{(4)}(x) = -96(1+2x)^{-4},$$
$$f^{(4)}(0) = -96$$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ...$$
$$+ \frac{f^{(n)}(a)}{n!}(x-a)^n + ...$$
$$\ln(1+2x) = 0 + 2x + \frac{-4}{2!}x^2 + \frac{16}{3!}x^3 + \frac{-96}{4!}x^4 + ...$$
$$= 2x - 2x^2 + \frac{8}{3}x^3 - 4x^4 + ...$$