M242 Q11(c) (S. Zhang) (8 points). Name:______________

1. Find the limit by (1) L'Hopital rule, (2) Taylor series expansion.

$$\lim_{x\to 0} \frac{e^x - 1 - x - x^2}{\cos x - 1}$$

- **<u>ans:</u>** (1) 0/0.

$$\begin{aligned}\lim_{x\to 0} \frac{e^x - 1 - x - x^2}{\cos x - 1} &\overset{L'H}{=} \lim_{x\to 0} \frac{e^x - 1 - 2x}{-\sin x} \\ &\overset{L'H}{=} \lim_{x\to 0} \frac{e^x - 2}{-\cos x} \\ &= \frac{-1}{-1} = 1\end{aligned}$$

(2)

$$\begin{aligned}&\lim_{x\to 0} \frac{e^x - 1 - x - x^2}{\cos x - 1} \\ &= \lim_{x\to 0} \frac{(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...) - 1 - x - x^2}{(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ...) - 1} \\ &= \lim_{x\to 0} \frac{-\frac{x^2}{2} + ...}{-\frac{x^2}{2!} + ...} \\ &= \lim_{x\to 0} \frac{-\frac{1}{2} + ...}{-\frac{1}{2!} - ...} = \frac{-\frac{1}{2}}{-\frac{1}{2!}} = 1\end{aligned}$$